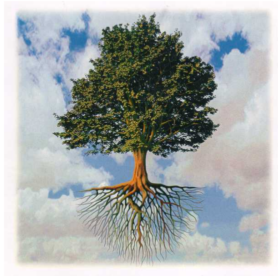

Project no. 265432

EveryAware

Enhance Environmental Awareness through Social Information Technologies

http://www.everyaware.eu

Seventh Framework Programme (FP7)

Future and Emerging Technologies of the Information Communication Technologies (ICT FET Open)

D5.1: Report on modelling social dynamics

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Executive Summary

The aim of this project is to raise the environmental awareness of people that in turn will trigger more sensible behaviors, so to pursue the ideal of a sustainable world. To this purpose it is crucial to monitor people's opinions in order to understand whether such behavior changes are under way. The procedure of collecting opinion data is appropriated to **WP1** by the use of modern ICT tools and to **WP2** by the use of web infrastructures. This work package (**WP5**) is devoted to the analysis of collected data by means of the methods of statistical physics.

The objective of this deliverable is to set up a theoretical basis for the analysis of the complex data that will be collected by the aforementioned work packages. The study of opinion dynamics has committed lots of scientists in the past. Substantial part of the present report has therefore to deal with a review of selected models proposed in the literature. Some of these models will be used to extract relevant informations from the data set that we shall collect in the near future.

A brand new opinion dynamics model that takes into account the effect of the media in the population as well as the effect of disagreement between peers, will also be described. In this model, we introduce an internal probability of individuals to choose between several discrete options, with both attractive and repulsive dynamics. An important feature of the model is the ability to introduce modulated external information, so that more possibilities can be promoted (e.g. by mass media). It will be shown, by numerical simulations, that an extreme information causes fragmentation and has limited success, while moderate information causes cohesion and has a better success in attracting individuals. We draw a parallel with the success of marketing or election campaigns, where coalitions and milder messages are more effective. Additionally, information success is maximised when individuals do not interact too much with the information, showing the importance of the social effect in information spreading.

Finally, a short preliminary study on user awareness on noise pollution based on data gathered with the **Widenoise** application will be discussed. We shall show how the repeated use of the Widenoise application increases users awareness on acoustic pollution. In fact, the first samples collected by users, show a guessed perceived sound level rather underestimating the actual measured value. The estimates become more accurate with increasing application usage so that, users succesfully train themselves to act as "reliable" acoustic sensors.

Outline of the document

After a brief overview of the subject in Chapter 1, in Chapter 2 we shall review existing models in opinion dynamics together with their present practical applications. In Chapter 3 we shall introduce our model of opinion dynamics bearing the ingredients of disagreement and modulated information. Finally, in Chapter 4 we report on a preliminary study about user awareness on acoustic pollution carried on with the Widenoise application. The full study on emerging awareness is scheduled at month 36.

Dissemination of the Results

The results on opinion dynamics presented in this deliverable have been submitted for publication in the *Journal of Statistical Mechanics* and were presented at the European Conference on Complex Systems (ECCS), September 2012, Brussels, within the workshop *Cultural and opinion dynamics: modelling, experiments and challenges for the future*.

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Chapter 1

Overview

Given its success and its very general conceptual framework, in recent years there has been a trend toward applications of statistical physics to interdisciplinary fields as diverse as biology, medicine, information technology, computer science, etc. In this context, physicists have shown a rapidly growing interest for a statistical physical modelling of fields patently very far from their traditional domain of investigations [Chakrabarti et al., 2006; Stauffer et al., 2006]. In social phenomena the basic constituents are not particles but humans and every individual interacts with a limited number of peers, usually negligible compared to the total number of people in the system. In spite of that, human societies are characterized by stunning global regularities [Buchanan, 2007]. There are transitions from disorder to order, like the spontaneous formation of a common language/culture or the emergence of consensus about a specific issue. There are examples of scaling and universality. These macroscopic phenomena naturally call for a statistical physics approach to social behaviour, i.e., the attempt to understand regularities at large scale as collective effects of the interaction among single individuals, considered as relatively simple entities.

It may be surprising, but the idea of a physical modelling of social phenomena is in some sense older than the idea of statistical modelling of physical phenomena. The discovery of quantitative laws in the collective properties of a large number of people, as revealed for example by birth and death rates or crime statistics, was one of the factors pushing for the development of statistics and led many scientists and philosophers to call for some quantitative understanding (in the sense of physics) on how such precise regularities arise out of the apparently erratic behaviour of single individuals. Hobbes, Laplace, Comte, Stuart Mill and many others shared, to a different extent, this line of thought [Ball, 2004]. This point of view was well known to Maxwell and Boltzmann and probably played a role when they abandoned the idea of describing the trajectory of single particles and introduced a statistical description for gases, laying the foundations of modern statistical physics. The value of statistical laws for social sciences has been foreseen also by Majorana in his famous tenth article [Majorana, 1942, 2005]. But it is only in the past few years that the idea of approaching society within the framework of statistical physics has transformed from a philosophical declaration of principles to a concrete research effort involving a critical mass of physicists. The availability of new large databases as well as the appearance of brand new social phenomena (mostly related to the Internet) and the tendency of social scientists to move toward the formulation of simplified models and their quantitative analysis, have been instrumental for this change.

Agreement is one of the most important aspects of social group dynamics. Everyday life presents many situations in which it is necessary for a group to reach shared decisions. Agreement makes a position stronger, and amplifies its impact on society. The dynamics of agreement/disagreement among individuals is complex, because the individuals are. Statistical physicists working on opinion dynamics aim at defining the opinion states of a population, and the elementary processes that determine transitions between such states. The main question is whether this is possible and whether this approach can shed new light on the process of opinion formation.

In any mathematical model, opinion has to be a variable, or a set of variables, i.e., a collection of numbers. This may appear too reductive, thinking about the complexity of a person and of each individual position. Everyday life, on the contrary, indicates that people are sometimes confronted with a limited number of positions on a specific issue, which often are as few as two: right/left, Windows/Linux, buying/selling, etc. If opinions can be represented by numbers, the challenge is to find an adequate set of mathematical rules to describe the mechanisms ruling their evolution and changes.

The first model of opinion dynamics designed by a physicist was proposed in [Weidlich, 1971]. Later on, the Ising model was adapted to an opinion dynamics model [Galam and Moscovici, 1991; Galam et al., 1982]. Briefly, the spin-spin coupling represented the pairwise interaction between agents, the magnetic field the cultural majority or propaganda, the temperature a sort of statistical disorder. Moreover, individual fields are introduced that determine personal preferences toward either orientation. Depending on the strength of the individual fields, the system may reach total consensus toward one of the two possible opinions, or a state where both opinions coexist. The plain Ising model is particularly attractive if viewed in the frame of our project, since it foresees a phase transition whenever the system is not too disordered (i.e. below the transition temperature), irrespective to the strength of a not null propaganda. This behaviour, if true, could be the key toward a global social behavioural change leveraged by an enhanced awareness on the environmental issues of our daily world.

Unfortunately, the Ising model is by far too simple. In our real life we do not interact with a few adjacent neighbours, rather we are part of a complex network of social contacts. Moreover, because of the human essence, we are not identical to each other (that is why one has to introduce individual fields for the Ising model to make sense) and the interaction between peers may depend on the their degree of mutual agreement. Also, real interaction between individuals in society does not only involve agreement, but disagreement is extremely important [Huckfeldt et al., 2004]. Furthermore, apart in few well justified cases, generally speaking opinions are not limited to a binary variable, but are rather best represented by a (dynamical) continuum of values, as for instance described in [Deffuant et al., 2000]. Finally, different opinion coexist in our minds. These opinions interact with each other and form a complex multi-dimensional space. Well established attempts to cope with this multi-dimensionality start with [Axelrod, 1997].

In the last decade, physicists have started to work actively in opinion dynamics, and many models have been designed (an extensive review of these can be found in [Castellano et al., 2009a]). In the following, we present some of these models and their latest developments (Chapter 2), with a review of modelling the effect of external information on opinion dynamics in Section 2.3. We then introduce a novel method of modelling opinion dynamics, using disagreement and modulated external information (Chapter 3).

Chapter 2

Existing models for opinion dynamics and applications

2.1 One-dimensional models

2.1.1 Discrete opinions

The voter model

The voter model owes its name to the very natural interpretation of its rules in terms of opinion dynamics. Because of its extremely simple definition, however, the model has been thoroughly investigated also in fields quite far from social dynamics, like probability theory and population genetics. Voter dynamics was first considered in [Clifford and Sudbury, 1973] as a model for the competition of species and named "voter model" in [Holley and Liggett, 1975]. It has soon become popular because, despite being a rather crude description of any real process, it is one of the very few non-equilibrium stochastic processes that can be solved exactly in any dimension [Redner, 2001].

Its definition is extremely simple: each agent is endowed with a binary variable $s = \pm 1$. At each time step an agent i is selected along with one of its neighbours j and $s_i = s_j$, i.e., the agent takes the opinion of the neighbour. This update rule implies that agents imitate their neighbours. They feel the pressure of the majority of their peers only in an average sense: the state of the majority does not play a direct role and more fluctuations may be expected with respect to the zero-temperature Glauber dynamics. Bulk noise is absent in the model, so the states with all sites equal (consensus) are absorbing. Starting from a disordered initial condition, voter dynamics tends to increase the order of the system, as in usual coarsening processes [Scheucher and Spohn, 1988]. The question is whether full consensus is reached in a system of infinite size.

For the Ising model, macroscopically, the dynamic driving force towards order is surface tension. Interfaces between domains of opposite magnetization cost in terms of energy and their contribution can be minimized by making them as straight as possible. This type of ordering is often referred to as curvature-driven. The presence of surface tension is a consequence of the tendency of each spin to become aligned with the majority of its neighbours. However, in the Voter model, the majority does not play a direct role. A look at the patterns generated in two-dimensional lattices (Fig. 2.1) indicates that domains grow but interfaces are very rough, at odds with usual coarsening systems [Bray, 1994]. More physical insight is provided by considering a droplet of up spins surrounded by negative spins [Dornic et al., 2001]. The Cahn-Allen theory for curvature-driven coarsening [Bray, 1994] predicts in $d = 2$ a linear decay in time of the droplet area, the rate being proportional to surface tension. In the voter model instead, the interface of the droplet roughens but its radius remains statistically unchanged [Dall'Asta and Castellano, 2007; Dornic et al., 2001], showing that no surface tension is present (Fig. 2.1).

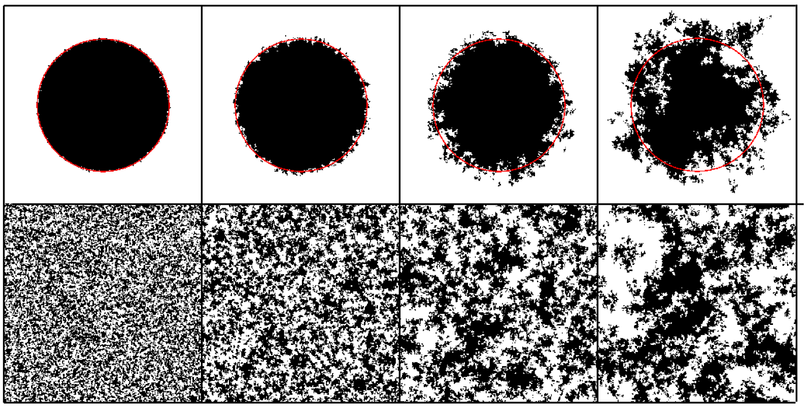

Figure 2.1: Evolution of a two-dimensional voter model starting from a droplet (top) or a fully disordered configuration (bottom). From [Dornic et al., 2001].

Considering a d-dimensional hypercubic lattice, in an infinite system consensus is reached only if $d \leq 2$, while in any dimension consensus is invariably reached asymptotically if the system is finite. The time T_N needed depends on the system size N [Cox, 1989]: $T_N \sim N^2$ for $d = 1$, $T_N \sim N \ln N$ for $d = 2$, while $T_N \sim N$ for $d > 2$. It is worth remarking that the way consensus is reached on finite systems has a completely different nature for $d \leq 2$, where the system coherently tends towards order by coarsening, and for $d > 2$, where consensus is reached only because of a large random fluctuation.

When considering disordered topologies different ways of defining the voter dynamics, that are perfectly equivalent on regular lattices, give rise to nonequivalent generalizations of the voter model. When the degree distribution is heterogeneous, the order in which a site and the neighbour to be copied are selected does matter, because high-degree nodes are more easily chosen as neighbours than low-degree vertices.

The most natural generalization (*direct voter* model) is to pick up a site and make it equal to one of its neighbours. In this way one of the fundamental properties of the voter model, conservation of the global magnetization, is violated [Suchecki et al., 2005; Wu et al., 2004]. To restore conservation a *link-update* dynamics must be considered [Suchecki et al., 2005]: a link is selected at random and then one node located at a randomly chosen end is set equal to the other. If instead one chooses first a node and copies its variable to a randomly selected neighbour one obtains the *reverse (or invasion) voter* dynamics [Castellano, 2005].

On highly heterogeneous substrates these different definitions result in different behaviours. The mean consensus time T_N has been computed in [Sood and Redner, 2005; Sood et al., 2007] for the direct voter dynamics on a generic graph, by exploiting the conservation of a suitably defined degree-weighted density ω of up spins,

$$T_N(\omega) = -N\frac{\mu_1^2}{\mu_2}\left[(1-\omega)\ln(1-\omega) + \omega \ln \omega)\right], \quad (2.1)$$

where μ_k is the k-th moment of the degree-distribution. For networks with scale-free distributed degree (with exponent γ), T_N scales then as N for $\gamma > 3$ and sublinearly for $\gamma \leq 3$, in good agreement with numerical simulations [Castellano et al., 2005; Sood and Redner, 2005; Suchecki et al., 2005]. The same approach gives, for the other versions of voter dynamics on graphs, a linear dependence of the consensus time on N for link-update dynamics (independent of the degree distribution) and $T_N \sim N$ for any $\gamma > 2$ for the reverse-voter dynamics, again in good agreement with simulations [Castellano, 2005; Sood et al., 2007]. A general analysis of voter-like dynamics on generic topologies is presented in [Baxter et al., 2008], with particular reference to applications in population genetics and biodiversity studies.

A recent development involves using power-law intervals between interactions [Takaguchi and Masuda, 2011], as opposed to the exponential interval distribution in the original model. The analysis is performed on different network topologies, i.e. ring, complete graphs and regular random graphs. In general, power-law intervals slow down the convergence time, with small, if no differences, seen for the complete graphs, medium for regular random graphs and large for the ring. In [Benczik et al., 2009], the voter model is analysed on random networks, where links are rearranged in an adaptive manner, based on agent similarity (links with agents not sharing the opinion are dropped in favour of new connections to individuals having the same opinion). They show analytically that in finite systems consensus can be reached, while in infinite systems metastable states can persist for an infinitely long time. A different analysis on a *directed* adaptive network has been proposed in [Zschaler et al., 2011] where link directionality is shown to induce an early fragmentation in the population.

A non-linear extension of the model is introduced in [Yang et al., 2011]. This allows agents to select their opinion based on their neighbours using a parameter α which controls the herding effect. The probability that an agent adopts opinion $+1$ is:

$$P(+1) = \frac{n_+^\alpha}{n_+^\alpha + n_-^\alpha} \tag{2.2}$$

Where n_+ (n_-) is the number of neighbours holding an opinion $+1$ (-1). For $\alpha = 1$ the original voter model is retrieved, while for large α a model similar to the majority rule (next paragraph) is obtained. Convergence time is analysed depending on α, and it is shown that a minimum is obtained for moderate values of α. For extremely low values, large clusters form slowly, while for very large values, large opinion clusters take long to merge. This indicates that in order to accelerate consensus, the local majority opinion should not be strictly followed, but this should be followed in a moderate way. The optimal α decreases with system size. This holds for a few network types analysed: regular lattices, Erdos-Renyi random graphs, scale-free and small-world networks. For the complex networks, the minimum α is also shown to increase with the network connectivity (average degree of the nodes).

The majority rule model

In a population of N agents, endowed with binary opinions, a fraction p_+ of agents has opinion $+1$ while a fraction $p_- = 1 - p_+$ opinion -1. For simplicity, suppose that all agents can communicate with each other, so that the social network of contacts is a complete graph. At each iteration, a group of r agents is selected at random (discussion group): as a consequence of the interaction, all agents take the majority opinion inside the group (Fig. 2.2). This is the basic principle of the majority rule (MR) model, which was proposed to describe public debates [Galam, 2002].

The group size r is not fixed, but is selected at each step from a given distribution. If r is odd, there is always a majority in favour of either opinion. If r is even, instead, there is the possibility of a tie, i.e., that either opinion is supported by exactly $r/2$ agents. In this case, one introduces a bias in favour of one of the opinions, say $+1$, and that opinion prevails in the group. This prescription is inspired by the principle of social inertia, for which people are reluctant to accept a reform if there is no clear majority in its favour [Friedman and Friedman, 1984]. Majority rule with opinion bias was originally applied within a simple model describing hierarchical voting in a society [Galam, 1986, 1990, 1999, 2000].

Defined as p_+^0 the initial fraction of agents with the opinion $+1$, the dynamics is characterized by a threshold p_c such that, for $p_+^0 > p_c$ ($p_+^0 < p_c$), all agents will have opinion $+1$ (-1) in the long run. The time to reach consensus (in number of updates per spin) scales like $\log N$ [Tessone et al., 2004]. If the group sizes are odd, $p_c(r) = 1/2$, due to the symmetry of the two opinions. If there are groups with r even, $p_c < 1/2$, i.e., the favoured opinion will eventually be the dominant one, even if it is initially shared by a minority of agents.

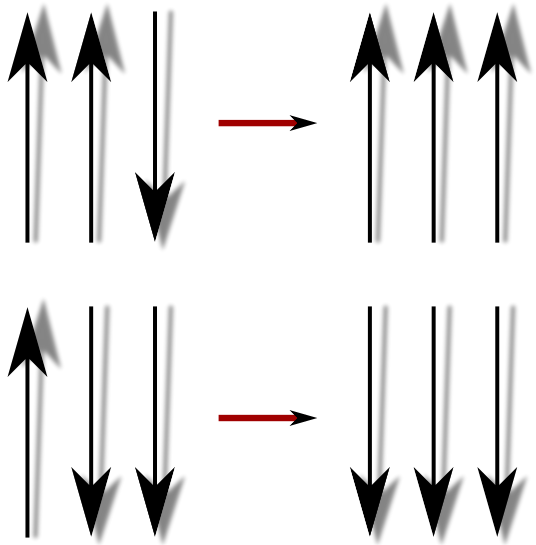

Figure 2.2: Majority Rule model. The majority opinion inside a discussion group (here of size three) is taken by all agents.

A full review of extensions and application of the majority rule model can be found in [Galam, 2008b]. Recent extensions have been used to explain results of public debates on different issues such as global warming, evolution theory, H1N1 pandemic [Galam, 2010a]. These include two types of agents, floater and inflexible, where inflexible agents do not change their opinion. It is shown that, for the case where not enough scientific data is available, the inflexible agents are those that drive the result of the debate. Hence, a strategy for winning a debate is the acquisition of as many inflexible agents as possible. Also, the analyses indicate that a fair discourse in a public debate will most likely lead to losing, while exaggerated claims are very useful for winning. Similar results are presented in [Galam, 2008a], where contrarians, i.e. agents who take the minority opinion of a group, are also introduced.

The same issue of public debates has been analysed with a different variation of the model [Galam, 2010b]. Here, collective beliefs are introduced as an individual bias to select one or the other opinion, in case of a tie in voting. Here only pair interactions are analysed. The study shows that collective beliefs are very important in determining the results of the debate, and again, a winning strategy is acquiring inflexible agents, which may mean using overstated or exaggerated statements. A similar model has been also applied to explain the formation of bubble crashes in the financial market [Galam, 2011]. Agents decide to sell or buy depending on the majority rule and the collective beliefs in case of tie. The model shows that it is the collective beliefs that determine a discrepancy between the real and the market value of an asset, which in turn generates crashes. If the collective beliefs are balanced, or ties do not appear (by using odd-sized groups), these crashes do not appear.

An application of a similar model, entitled majority vote model, to model tax evasion dynamics is presented in [Lima, 2012]. Here, $+1$ represents and honest individual, while -1 an individual evading tax. Individuals change their opinion with a probability which depends on the average of all of their neighbours:

$$P('flip') = \frac{1}{2}\left[1 - (1-2q)\sigma_i sign(\sum_{n \in N(i)} \sigma_n)\right] \tag{2.3}$$

Here, σ_i is the current opinion of individual i, $N(i)$ is the set of neighbours of i, while q is a noise parameter. The model is also introduced an audit procedure. When an agent chooses to evade tax, a punishment is imposed with probability p, consisting in forcing the agent to be honest for a number of k population updates. Different network topologies are analysed: square lattice, Barabasi-Albert and Honisch-Stauffer. Numerical results show that without punishment, tax evasion fluctuates, reaching at times very high levels. The introduction of audit, even at very low levels, is shown to reduce drastically the percentage of agents choosing to avoid tax.

Social impact and the Sznajd model

Interactions and opinion formation, with their complex underlying features, have been long analysed by social scientists, and theories devised to explain them. One example is social impact theory [Latane, 1981], which states that the impact of a group of people on an individual depends mainly on three factors: their number, their distance and their strength. A first application of this theory to build a dynamical model of opinion formation has been introduced in [Lewenstein et al., 1992; Nowak and Lewenstein, 1996]. This uses cellular automata to model individuals which hold one of two opinion values $\sigma_i = \pm 1$. They are placed within a network, which accounts for the spatial factor, i.e. the distance d between individuals. Individual strength is represented by two variables: persuasiveness (how much is an agent able to influence another) and supportiveness (how much an agent supports the opinion they hold in their neighbourhood). Social impact on individual i is then computed as a weighted sum of the persuasiveness of other agents holding a different opinion and the supportiveness of agents holding the same opinion :

$$I_i = I_p \left(\sum_j \frac{t(p_j)}{g(d_{ij})} (1 - \sigma_i \sigma_j) \right) - I_s \left(\sum_j \frac{t(s_j)}{g(d_{ij})} (1 + \sigma_i \sigma_j) \right) \quad (2.4)$$

Here d_{ij} is the distance between agents i and j (which can be defined depending on the network type used), $g()$ is a decreasing function of d_{ij} and $t()$ is a strength scaling function. Thus, the updating rule for opinion of agent i is:

$$\sigma_i' = -sign(\sigma_i I_i + h) \quad (2.5)$$

where h is a noise factor. The model was shown to lead to spatially localised opinion clusters, where minority clusters are facilitated by the existence of strong individuals supporting the weaker ones. This holds for a variety of social network topologies: fully connected graph, hierarchical networks, strongly diluted networks and Euclidean space.

Another recent model employing the theory of social impact is the Sznajd model [Sznajd-Weron and Sznajd, 2001]. This is a variant of spin model, on a one dimensional lattice, that takes into account the fact that a group of individuals with the same opinion can influence their neighbours more than one single individual. The proximity factor is also taken into account, by considering neighbouring agents on the lattice. However, the strength of individuals, a third factor mentioned in the theory of social impact, is not present. Each agent has an opinion $\sigma_i = \pm 1$. At each time step, a pair of neighbouring agents is selected and, if their opinion coincides, all their neighbours take that opinion. Otherwise, the neighbours take contrasting opinions. The model has been shown to converge to one of the two agreeing stationary states, depending on the initial density of up-spins (transition at 50% density). Versions on a two dimensional lattice have also been studied, with four neighbours (a plaquette) having to agree in order to influence their other 8 neighbours [Stauffer et al., 2000], Figure 2.3. Extensions to a third option (centrist/indifferent) have been also studied [Baker and Hague, 2008; Malarz, 2009].

A different extension is the introduction of "social temperature" [Lama et al., 2005]. Here the original rules of the Sznajd model are applied with probability p, i.e. all neighbours take the opinion value of the plaquette, in case they agree. With probability $1 - p$ the agents take the opposite value than dictated by the original Sznajd rules. This results in disagreement by some individuals who choose to be or not to be contrarians at each update. Importantly, disagreement is not a fixed attribute of

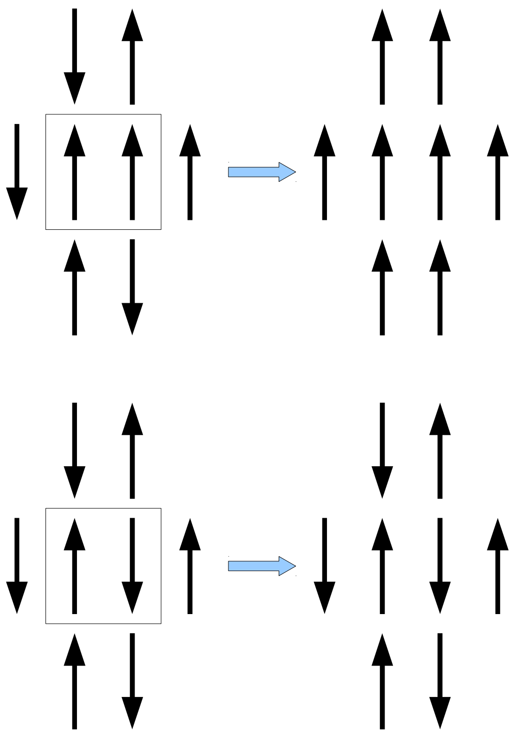

Figure 2.3: Sznajd model. A pair of neighbouring agents with the same opinion convince all their neighbours (top), while they have no influence if they disagree (bottom).

the individuals, but varies in time. It was shown that over a critical threshold for p, the behaviour of the original model is conserved, i.e. all individuals agree to one opinion. Under this threshold the system remains in a disordered state with magnetisation (defined as $\frac{\sum_{i=1}^{N} \sigma_i}{N}$)close to 0.

A recent study of disagreement in the Sznajd model in one dimension is [Kondrat and Sznajd-Weron, 2010], where conformist (agreement) and anti-conformist (disagreement) reactions appear. Specifically, the model is introduced a parameter p which defines the probability that, when two neighbours hold the same opinion, a third neighbour, that previously held the same opinion, will take the opposite position. If the third neighbour did not share the opinion of the initial pair, then they take that opinion, as in the original Sznajd model. It is shown that for low anti-conformity, consensus can be reached, and spontaneous shifts in the entire population between ± 1 appear. On the other hand, high anti-conformity results in oscillations of the magnetisation level around 0, without reaching ± 1. The same model has been applied on complete graphs [Nyczka et al., 2012a]. Here, it was shown (both numerically and analytically) that the reorientations for low anti-conformism (p) appear now between two magnetisation states $\pm m$ instead of ± 1.

Agent independence (as opposed to disagreement) is studied in [Sznajd-Weron et al., 2011]. Independence means that a neighbouring agent can choose not to follow an agreeing plaquette, with probability p. In this case, they can flip their opinion with probability f, described as agent flexibility. The model is analysed on one and two-dimensional lattices and on a complete graph. Independence is shown to favour coexistence of the two opinions in the society, with the majority being larger for small independence levels (p).

The Sznajd model with reputation, on a 2-D lattice, has been also analysed [Crokidakis and De Oliveira, 2011], where each agent has a reputation value associated. The agent plaquette can influence the neighbours, with probability p, only if they agree and they have an average reputation larger than the neighbours. Reputations also evolve, i.e. they increase if the plaquette influences a neighbour and decrease otherwise. The model is shown to lose the phase transition for $p < p_c \sim 0.69$, when some agents preserve a non-majoritary opinion.

An analysis of the Sznajd model on an Erdos-Renyi random graph with enhanced clustering is presented in [Malarz and Kulakowski, 2008], where the model is shown to not reach full consensus, unlike the original model. The connection of a modified version of the model, which includes bounded confidence and multiple discrete opinions, with graph theory is discussed in [Timpanaro and Prado, 2012].

The q-voter model

In [Castellano et al., 2009b] the non-linear q-voter model is introduced, as a generalisation of discrete opinion models. Here, N individuals in a fully connected network, hold an opinion ± 1. At each time step, a set of q neighbours are chosen and, if they agree, they influence one neighbour chosen at random, i.e. this agent copies the opinion of the group. If the group does not agree, the agent flips its opinion with probability ϵ. The voter and Sznajd models and many of their extensions are special cases of this more recent model. Analytical results for q<=3 validate the numerical results obtained for the special case models, with transitions from a ordered phase (small ϵ) to a disordered one (large ϵ). For q>3, a new type of transition between the two phases appears, which consist of passing through an intermediate regime where the final state depends on the initial condition.

In [Nyczka et al., 2012b] the q-voter model is analysed for non-conformity and anti-conformity with the aim to compare the two types of dynamics. Nonconformity implies that some agents, regardless of what the influencing group's opinion is, will decide to flip their opinion with probability p. Anti-conformity means that some agents will not follow the opinion of the group, but the opposite one, with probability p. The comparison shows important difference between the two types of dynamics, although they appear to be very similar. In the case of anti-conformism, the critical value p_c for the order-disorder phase transition is shown to increase with q, while for non-conformism, this decreases with q.

2.1.2 Continuous opinions

Deffuant-Weisbuch

The Deffuant-Weisbuch model [Deffuant et al., 2000] uses a continuous opinion space, where each individual out of a population of N can take an opinion value $x_i \in [-1, 1]$. Two individuals interact if their opinions are close enough, i.e. $|x_i - x_j| < d$, with d a bounded confidence parameter. In this case, they get closer to one another by an amount determined by the difference between them and a convergence parameter μ:

$$x_i = x_i + \mu(x_j - x_i) \tag{2.6}$$

The population was shown to display convergence to one or more clusters (c) depending on the value of the bounded confidence parameter ($c \approx \lfloor \frac{1}{2d} \rfloor$ [Carletti et al., 2006]). Parameters μ and N (population size) determine the convergence speed and the width of the distribution of final opinions.

The Deffuant-Weisbuch model has received a lot of attention in the literature, with several recent analysis and extensions. For instance, [Weisbuch et al., 2002] discusses heterogeneous and adaptive confidence thresholds on 2D lattices. In [Gomez-Serrano et al., 2010], analytical results are provided, showing that in the limit of time $t \to \infty$, the population forms a set of clusters too far apart to interact, at a distance larger than d, after which agents in individual clusters converge to the cluster's barycentre. When $N \to \infty$, the opinion evolution is shown to be equivalent to a nonlinear Markov process, which proves the “propagation of chaos" for the system. This means that, as the system becomes infinite in size, an opinion evolves under the influence of opinions selected independently from the opinion process, at a rate given by the limit of the rate at which agents interact in the finite system.

The original model is based on agreement dynamics, i.e. if individuals are too different, they do not interact. However, disagreement dynamics are well known to appear in real situations. Hence, in [Kurmyshev et al., 2011], partial contrarians were included, which are agents that can disagree (i.e. change their opinion in the opposite direction) with individuals that think differently. The society is mixed with the two types of agents, and it is shown that dynamics change depending on the amount of individuals that can disagree. Depending on the value of the bounded confidence parameter, one, two or more clusters can be observed, similar to the original Deffuant-Weisbuch, but bifurcation patterns are different. For a large number of contrarians, the number of clusters decreases as the confidence increases, but clusters become more different. For a smaller number of contrarians, on the other hand, clusters also become closer when they are fewer. This shows that contrarians favour a more determined fragmentation, i.e. not only the number of clusters, but also the distance between clusters increases. Also, the new type of agents increases the time required to reach a final frozen state. A similar approach can be found in [Huet et al., 2008], where the 2-D Deffuant model with disagreement is analysed, and shown to favour extremist clusters.

A different extension of the model is to consider the bounded confidence parameter as an attribute of the individuals, hence different for each. In [Lorenz, 2010] heterogeneous bounds of confidence are shown to enhance the chance for consensus, since close-minded individuals can be influenced by the more open-minded ones (this extension has been also applied to the Hegselmann-Krause model described in the next section). On the same lines, [Gargiulo and Mazzoni, 2008] devises a method of computing the bounded confidence threshold based on the current individual opinion, to obtain less confidence for extremists:

$$d_i = 1 - \alpha|x_i|, \tag{2.7}$$

where α controls the tolerance rate. The update rule is also changed so that extremists change their opinion less:

$$x_i = x_i + d_i(x_j - x_i)/2 \tag{2.8}$$

Additionally, the social network is determined at the beginning depending on how extreme are individual opinions (extremists interact only with similar individuals, while moderated individuals can interact with a wider range, based on a segregation parameter β). Under these new conditions, it is shown that opinions converge to one large cluster when α is very small or β is very large, with some small coexisting extreme clusters, while pluralism is conserved only when extremist clusters are connected enough to continue to communicate to others (large α and β).

Further, in [Gandica et al., 2010] an analysis of the Deffuant-Weisbuch model on scale free directed social networks is presented, and the average number of final opinions is shown to be larger, when compared to undirected networks, for high bounded confidence parameter d and smaller for low d. Also, an analysis on an adaptive network is presented in [Gargiulo and Huet, 2010].

The Hegselmann-Krause model

The model proposed in [Hegselmann and Krause, 2002] (HK) is similar to that of Deffuant. Opinions take real values in an interval, say $[0, 1]$, and an agent i, with opinion x_i, interacts with neighbouring agents whose opinions lie in the range $[x_i - \epsilon, x_i + \epsilon]$, where ϵ is the uncertainty. The difference is given by the update rule: agent i does not interact with one of its compatible neighbours, like in Deffuant, but with all its compatible neighbours at once. Deffuant's prescription is suitable to describe the opinion dynamics of large populations, where people meet in small groups, like pairs. In contrast, HK rule is intended to describe formal meetings, where there is an effective interaction involving many people at the same time.

On a generic graph, HK update rule for the opinion of agent i at time t reads:

$$x_i(t+1) = \frac{\sum_{j:|x_i(t)-x_j(t)|<\epsilon} a_{ij} x_j(t)}{\sum_{j:|x_i(t)-x_j(t)|<\epsilon} a_{ij}}, \tag{2.9}$$

where a_{ij} is the adjacency matrix of the graph. So, agent i takes the average opinion of its compatible neighbours. The model is fully determined by the uncertainty ϵ, unlike Deffuant dynamics, for which one needs to specify as well the convergence parameter μ.

The dynamics develops just like in Deffuant, and leads to the same pattern of stationary states, with the number of final opinion clusters decreasing if ϵ increases. In particular, for ϵ above some threshold ϵ_c, there can only be one cluster. Recently, an in-depth analysis of clustering patterns, depending on ϵ has been performed [Slanina, 2010], and it was shown that there are genuine dynamical phase transitions between k and $k+1$ clusters, and that around critical values of ϵ, the dynamics slows down.

A further study on the clustering patterns [Blondel et al., 2009] proved analytically that the population in the HK model with real opinions (not restricted to interval [0,1], but to [0,L]) and $\epsilon = 1$ converges always to clusters that are at distance larger than 1, and provided calculation of lower bounds of inter-cluster distance both for finite size and a continuum of agents. The continuum version of the model considers individuals indexed by the real interval $I = [0, 1]$, which have opinions in interval $[0, L]$. Hence, for $\alpha \in [0, 1]$, $x_t(\alpha) \in [0, L]$ is the opinion of individual α at time t. Defining $C_x = \{(\alpha, \beta) \in I^2 / |x(\alpha) - x(\beta)| < 1\}$, the update rule becomes:

$$x_{t+1}(\alpha) = \frac{\int_{\beta:(\alpha,\beta)\in C_{x_t}} x_t(\beta) d\beta}{\int_{\beta:(\alpha,\beta)\in C_{x_t}} d\beta} \tag{2.10}$$

Proofs are given that during convergence, there is always a finite density of individuals between two clusters, which indicates that the model can never converge to an unstable equilibrium. This model is shown to be the limit of the original HK model, as the population size goes to infinity.

Additionally, in [Mirtabatabaei and Bullo, 2011], an analysis of the interaction network is performed. This is dynamic in the HK model, and evolves with the agent opinions, to reach a steady state where the network converges to a fixed topology, as demonstrated by [Mirtabatabaei and Bullo, 2011]. A different approach of devising analytical results for this model is by looking at the evolution of the distribution of opinions in the population, i.e. Eulerian HK model [Mirtabatabaei and Jia, 2012].

Other models

Apart from the above mentioned models, several agent-based approaches have been introduced, which share similarities with the Deffuant and Hegselmann-Krause models. In [Fent et al., 2007], a population of individuals with opinions in a continuous interval evolves based on in- and out-group interactions. Agents tend to be more similar to their in-group and more distant from their out-group, while being also reluctant to change opinion. Specifically, dynamics are determined by the objective of maximising a utility function, which includes the difference between agents and out-group, the similarity between them and in-group, and the closeness between their own opinion at time t and $t+1$. Simulation results show that when the out-group is small, the population reaches consensus to a mild opinion, for a medium out-group several clusters form, while a large out-group results in clusters where the two extreme opinions are acquired. In [Mas et al., 2010] a model of continuous opinions, balancing individualisation versus social integration, with adaptive noise, is introduced. Depending on the noise and individualisation levels, three states of the population can be obtained: consensus, individualism or preserved pluralism.

A different agent based modelling approach is [Mavrodiev et al., 2012], where the effect of social influence on the wisdom of crowds is analysed. The concept of wisdom of crowds means that the aggregated opinion of a group is closer to the truth than individual agent opinions. In this model, agents hold one continuous opinion on an issue, and interaction is modelled as the effect of the average opinion of peers. Simulation results show that the effect of social influence depends on the initial condition. Specifically, if the initial individual opinions are far from the truth, interaction has a beneficial effect, however, if they start close to the truth, social influence results in a decrease of the wisdom of crowds.

In [Acemoglu and Como, 2010], a continuous opinion model with Poissonian interaction intervals and stubborn agents is introduced. These agents do not change their opinion. The model was shown to generate continuous opinion fluctuation and disagreement in the population, i.e. consensus is never reached.

An approach different from those presented until now uses the Kuramoto model of coupled oscillators to describe opinion formation [Hong and Strogatz, 2011]. Two types of oscillators are considered, corresponding to agents which agree or disagree to others. Disagreeing oscillators are negatively coupled to the mean field. The paper shows that, even when oscillators have the same frequency, the introduction of disagreement leads to appearance of opposite clusters, travelling waves or complete incoherence.

2.1.3 Hybrid models

The CODA model

Continuous Opinions and Discrete Actions (CODA) are used to model the degree of acquiring a certain discrete opinion. The original model [Martins, 2007] considered two opinions +1 and -1. Individuals are represented on a square lattice by a continuous probability p_i showing the extent of agreement to opinion +1 (with $1-p_i$ corresponding to -1). Based on this, the choice of the discrete opinion σ_i is made, using a hard threshold:

$$\sigma_i = \mathrm{sign}\,(p_i - 0.5). \tag{2.11}$$

Individuals see only the discrete opinions of others, σ_i, and change the corresponding p_i based on their neighbours, using a Bayesian update rule, which favours agreement to the neighbours. This maintains the discrete public dynamics, and introduces both a means to quantify the extent of adhesion to one opinion and a memory effect (individuals do not jump directly from -1 to +1, but change their opinions continuously). The model is applied both to the Voter model of interaction, i.e. one agent interacts with one neighbour at each step, and to the Sznajd model, i.e. two neighbouring agents influence the rest of their neighbours. For both cases, the emergence of extremism even in societies of individuals that start with mild opinions at the beginning is shown. Relatively stable domains are formed within the population, which exhibit small changes after they are established. Disagreement dynamics are introduced in the model in [Martins and Kuba, 2010], by considering part of the population as contrarians (they always disagree with their peers). This has been shown to reduce agreement in the population, but at the same time to discourage extremist opinions, compared to the original model.

The model was also analysed under the assumption of migration in social networks [Martins, 2008], where each individual is allowed to change position, a mechanism shown to reduce the amount of extremism observed, yielding one cluster in the end. Further, in [Martins, 2009], a third opinion is introduced, either as 'undecided' (if p_i is close to 0.5) or a real alternative (usage of three probability values, p_i,q_i,r_i, for the three available options). In the first case, a decrease in the amount of mild opinions is observed, but at the same time the level of extremism (the maximum absolute value of p_i) decreases. In the second case, there are two different analyses performed. When the third opinion is considered independent (i), the level of agreement is similar between the three options, with extremists for each. Here, for simplification, a set of assumptions about symmetry between the choices are made. When the third choice is a transition between the initial two (ii), a higher number of individuals adhering to the middle option is seen. An additional analysis [Martins, 2012] consisted of making agents also aware of the possible effect they have on the others, and discusses also the relation to other models in the literature.

2.2 Multi-dimensional models

2.2.1 Discrete opinions

The Axelrod model

The Axelrod model for culture dynamics [Axelrod, 1997] has been introduced to model culture formation based on two principles, the preference of individuals to interact with similar peers (homophily) and the increase in similarity after an interaction appears. N individuals are located on the nodes of a network (or on the sites of a regular lattice) and are endowed with F integer variables $(\sigma_1, \ldots, \sigma_F)$ that can assume q values, $\sigma_f = 0, 1, \ldots, q-1$. The variables are called cultural *features* and q is the number of the possible *traits* allowed per feature. They model the different "beliefs, attitudes and behaviour" of individuals. In an elementary dynamic step, an individual i and one of his neighbours j are selected and the overlap between them

$$\omega_{i,j} = \frac{1}{F} \sum_{f=1}^{F} \delta_{\sigma_f(i),\sigma_f(j)}, \tag{2.12}$$

is computed, where $\delta_{i,j}$ is Kronecker's delta. With probability $\omega_{i,j}$ the interaction takes place: one of the features for which traits are different $(\sigma_f(i) \neq \sigma_f(j))$ is selected and the trait of the neighbour is set equal to $\sigma_f(i)$. Otherwise nothing happens. It is immediately clear that the dynamics tends to make interacting individuals more similar, but the interaction is more likely for neighbours already sharing many traits (homophily) and it becomes impossible when no trait is the same. There are two stable configurations for a pair of neighbours: when they are exactly equal, so that they belong to the same cultural region or when they are completely different, i.e., they sit at the border between cultural regions.

Starting from a disordered initial condition (for example with uniform random distribution of the traits) the evolution on any finite system leads unavoidably to one of the many absorbing states, which belongs to two classes: the q^F ordered states, in which all individuals have the same set of variables, or the other, more numerous, frozen states with coexistence of different cultural regions.

It turns out that which of the two classes is reached depends on the number of possible traits q in the initial condition [Castellano et al., 2000]. For small q, individuals share many traits with their neighbours, interactions are possible and quickly full consensus is achieved. For large q instead, very few individuals share traits. Few interactions occur, leading to the formation of small cultural domains that are not able to grow: a disordered frozen state. On regular lattices, the two regimes are separated by a phase transition at a critical value q_c, depending on F (Fig. 2.4).

This model has been widely analysed after its introduction, and here we present the more recent investigations. Although most studies were numerical, some analytical proofs were provided in [Lanchier, 2010], where it is shown that for $F = q = 2$ the majority of the population forms one cluster, while a partial proof for the fact that, if $q > F$, the population remains fragmented, is provided. In [Barbosa and Fontanari, 2009], the dependence of the number of cultural clusters on the lattice area ($A = L^2$, where L is the dimension of the lattice) was analysed. They show that when $F \geq 3$ and $q < q_c$, a strange non-monotonical relation between the number of clusters and A exists. Specifically, the number of coexisting clusters decreases beyond a certain threshold of the area, in contrast with well known results for species-area relaxation, where the number of species increases with A. Outside these parameter values, however, the expected culture-area relaxation is observed. This is described by a curve that is steep at first (i.e. the number of clusters

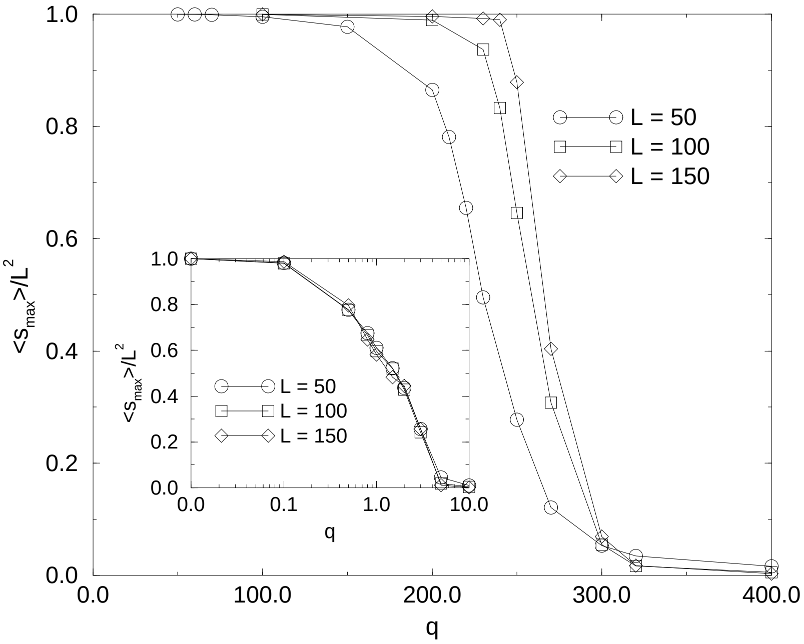

Figure 2.4: Axelrod model. Behaviour of the order parameter $\langle S_{max} \rangle / L^2$ vs. q for three different system sizes and $F = 10$. In the inset the same quantity is reported for $F = 2$. From [Castellano et al., 2000].

increases linearly with A) and then flattens when the maximum number of possible clusters (q^F) is reached.

Studies of the effect of cultural drift (external noise, i.e. some times agents choose to change one opinion randomly) [Klemm et al., 2003] showed that even a very small noise rate leads the system to agreement, while large noise favours fragmentation of cultures. Similar results were found in [De Sanctis and Galla, 2009], where an additional analysis of interaction noise (i.e. the probability to interact is modified by a small δ) showed small effects on the phase transition, but a reduction of relaxation times.

Disagreement dynamics have also been introduced [Radillo-Díaz et al., 2009], using a hard threshold for the overlap, under which individuals disagree. Disagreement causes individuals to change a common opinion on an issue, i.e. decrease their overlap. Two different versions of this model have been developed, one where all individuals can agree or disagree, and one where a fraction of individuals always agrees. In both cases, disagreement dynamics are shown to favour culture fragmentation.

In [Singh et al., 2011], committed individuals were introduced. These are individuals that do not change the opinion on one of the F issues. They are introduced as a fraction p of the whole population. Also, the social network evolves. The original Axelrod dynamics are changed. At each time step, an individual i is selected, and one of their neighbours j. If $o_{ij} < \phi$, Axelrod dynamics are followed, otherwise, the link between node i and j is removed and a random node is linked to i. The change in consensus time due to the introduction of committed individuals is analysed. For $p = 0$, consensus time grows exponentially with N, showing that rewiring impedes consensus in the population. When $p > 0$, consensus time is decreased. For $p < p_c \sim 0.1$, the exponential dependence is conserved, while for $p > p_c$, this becomes logarithmic in N. This shows that the introduction of committed individuals favours consensus in the population.

A study of the model on scale-free networks was presented in [Guerra et al., 2010]. This analyses individuals both at “microscopic” - individual feature value - and “macroscopic” level - entire vector of features. The aim is to study how cluster composition changes when moving between the two levels. They show that even when many individual features are common in the population, the global culture is still fragmented.

In [Banisch and Araújo, 2010], an application of the model to election data is presented, using a model version with only two possible discrete opinion values. Good similarity to election data is exhibited by the model during the transient stage of the dynamics, i.e. before opinions stabilise, when the vote distribution for each party follows the same scaling observed in real data.

2.2.2 Continuous opinions

An extension of continuous models like the Deffuant-Weisbuch to vectorial opinions (of size K) is presented in [Lorenz, 2007]. Here, the different vector elements are not independent, like in the Axelrod model, but they are constrained to a sum of 1. In this way, the different values could represent probabilities of choosing an opinion out of multiple possibilities on the same issue, or could model a resource allocation problem. The model applies bounded confidence, by using the Euclidean distance between two individuals (d_{ij}). Two model versions are analysed, following Deffuant-Weisbuch and Hegselmann-Krause dynamics. For the former, individuals interact if $d_{ij} < \varepsilon$, when one of the peers takes the opinion given by the average between itself and the neighbour. Updating rules similar to the original Hegselmann-Krause model are also defined. The model is shown to converge to one or more clusters depending on ε and K. When the number of options K increases, the model is shown to obtain better agreement (large maximal component), but at the same time a larger number of small separate clusters. Also, when ε increases above a threshold, the population converges to one opinion. This threshold decreases with K.

In [Deffuant et al., 2012], continuous opinions are applied to model individuals' opinion about others and themselves, i.e. each individual holds a set of N opinions. An analysis of vanity and opinion propagation is performed, under the idea that opinions from highly valued individuals propagate more easily. For large vanity, individuals cluster in groups where they have a high opinion of themselves and other group members, and low opinions of peers external to their group. If vanity is lower, then some individuals gain high reputation, while most of the population have a low one. Situations with one or two agents dominating the others are exposed.

A different approach using continuous opinions and affinities between individuals is presented in [Carletti et al., 2011]. Each individual holds a real opinion $x_i \in [0,1]$ plus a set of affinities to all other agents, i.e. a real vector $\alpha_i \in [0,1]^{N-1}$. These are updated simultaneously during agent interaction. The bounded confidence concept from the Deffuant model is maintained, but the definition is changed to accommodate for affinity values between individuals. Specifically, even if the opinion of two agents are not close enough, if their affinity is high, then they can still interact. Affinities, on the other hand, decrease if individuals hold diverging opinion and increase when their positions are close. The update rules are thus:

$$x_i^{t+1} = x_i^t - \frac{1}{2}(x_i^t - x_j^t)\Gamma_1(\alpha_{ij}^t) \tag{2.13}$$

$$\alpha_{ij}^{t+1} = \alpha_{ij}^t + \alpha_{ij}^t(1-\alpha_{ij}^t)\Gamma_2(x_i^t - x_j^t) \tag{2.14}$$

where $\Gamma_1(\alpha) = \frac{\tanh(\beta_1(\alpha-\alpha_c))+1}{2}$ and $\Gamma_2(x) = -\tanh(\beta_2(|x|-d))$ are two activating functions that are reduced to step function when β_1 and β_2 are large enough. Parameters d and α_c are the confidence thresholds, i.e. affinity values increase if opinions are closer than d, while individuals interact if their affinity is larger than α_c. The model starts with random opinions and affinities, and is allowed to relax to a stable state. Affinities are then interpreted in terms of a weighted social network, with α_{ij} the weight of the link between agents i and j. The authors show that the network obtained display small-world properties and weak ties.

2.3 Effect of external information on opinion dynamics

The models we reviewed so far apply to situations in which consensus spreads or tries to spread among populations according to peer mutual interactions. There is no reservoir, to use a term coined in physics, with which or against which the population interacts. This limitation can be justified in few special cases, as for instance the spreading of dialects or regional behavioural habits, where the external pressure pushed on individuals comes from the interactions among the individuals themselves. On the other hand, we are nowadays bombarded by a huge amount of external information, "external" meaning here that such information comes from other sources than word of mouth. We live in a world where the mass media play a fundamental role. In order to understand whether it is feasible to achieve whatever behavioural changes in the population in response to given stimuli, we must consider models in which there is an external source of information. Some efforts in this direction have been made by the scientific community so far, however approaches are still limited to only a few of the models presented in the previous section. In the following paragraphs, we review the state of the art of opinion dynamics modelling with external sources of information flow, and in the following chapter we describe our novel approach to the problem.

2.3.1 One dimensional opinion

The effect of mass media has been studied for the Sznajd model on a square lattice [Crokidakis, 2011], by introduction of an external agent (media, having value e.g. $+1$). If four neighbours agree, then all their other neighbours switch to their opinion. If they do not then the neighbours take the media opinion with probability p. It was shown that the final state (either all spins up or down) depends on both the initial density of up-spins and on the value of p. The larger p, the smaller the initial density of up-spins has to be to ensure full agreement to the media. For $p \gtrsim 0.18$, the population always converges to the value of the information. In [Sznajd-Weron et al., 2008], an extension of Sznajd to three opinion states was applied to the mobile telecommunication market in Poland. The effect of media is introduced, i.e. an individual accepts the plaquette opinion with probability p, or the influence from media with probability $1 - p$. Media is represented as a set of probabilities to choose one of the options. The authors found that for low advertising, small companies are taken over by larger ones, as it happens in reality.

Effects of external information on the dynamics of the Deffuant model have been also investigated [Carletti et al., 2006]. All individuals are exposed to an external source of information O, which promotes a specific opinion. Every T generations, the entire population interacts with the information. These interactions follow the same rules as with other individuals: the opinion is updated only if the bounded confidence condition is met (see Equation 2.6 for details). Experiments were performed with $\mu = 0.5$. Dynamics were shown to depend on the value of the information, on T and on the parameter d from the original model. If the confidence is large enough so that the information can reach all individuals, the population converges to this. On the other hand, if confidence is extremely small, it is shown that full agreement with the information can be never reached. If neither of this applies, two types of dynamics are observed:

(i) In the case of extreme information (close to 0 or 1) and low confidence, T has to be in a fixed interval for the complete agreement to information to appear. Outside this interval, some individuals move away from the information forming an additional cluster. This shows that for extreme information to be efficient in a close-minded population, individuals need to be exposed to information often enough, but also need to interact to each other.

(ii) In case of mild information or large confidence, complete agreement is found only when T is larger than a threshold. This shows that individuals that do not access the information directly (because the confidence threshold is not met) can be influenced only if a large number of peer interactions are allowed before re-exposure to information. When the population does not converge to the information, still, large fractions of individuals form a cluster around the information value (minimum value over 0.5).

Another approach of analysing the effects of mass media in an extension of the Deffuant model is [Gargiulo et al., 2008], where, each generation, individuals interact with an external information x_I modulated by a parameter ϵ, the information strength:

$$x_i = x_i + \mu\epsilon d_i(x_I - x_i), \tag{2.15}$$

where d_i is defined as in Equation 2.7. For mild information (low ϵ), individual opinions move towards the value of x_I, however for strong information, an increasing number of antagonistic clusters emerge. This shows that aggressive media campaigns are risky and might result in the population not acquiring the information.

A different model similar to the Deffuant considers both disagreement and effects of mass media (external information) [Vaz Martins et al., 2010]. Here, disagreement is included as an attribute $w_{ij} \in -1, +1$ of the link between two individuals (some couples always agree, others always disagree), and opinions take values in interval $[0, 1]$. The interaction causes a change in the opinion value based on the type of link:

$$x_i = x_i + \mu w_{ij}(x_j - x_i) \tag{2.16}$$

Additionally, an external information source is considered, applied to all individuals after a specific number of updates. The introduction of repulsive links was shown to favour consensus with the external information.

In [Hegselmann and Krause, 2006], truth seekers are introduced into the Hegselmann-Krause model, i.e. individuals that take into account the value of the truth T. This can be interpreted as individuals who interact with experts, and is similar with the interaction to an external source of information. The opinion of an individual, upon interaction to a peer, changes as:

$$x_i(t+1) = \alpha_i T + (1 - \alpha_i) f_i(x(t)) \tag{2.17}$$

where $f_i(x(t))$ is the right term in equation 2.9, while α represents the disposition of individuals to seek the truth (which can be seen as the strength of the information). It is important to notice that the effect of the truth is not based on bounded confidence, i.e. it affects individuals with $\alpha \neq 0$ regardless of their opinions. Results show that even for small α (0.1) for all individuals, or if at least for half of the population $\alpha \neq 0$, the population converges to the truth, provided the truth is not extreme. If the truth is extreme (close to ± 1), and not all agents have $\alpha \neq 0$, some individuals remain far from truth. Large values of α may result in more individuals with $\alpha = 0$ to stay away from truth, which means that too strong information may have a disadvantageous effect. A further analysis of the model with truth seekers is presented in [Kurz, 2011], where it is proven analytically that all truth seekers (individuals with $\alpha \neq 0$) converge to the truth, even if there are agents that do not seek the truth.

2.3.2 Multi-dimensional opinion

The effect of mass media or propaganda for the Axelrod model has been widely studied, by introducing an external individual (information source) that can interact with the agents in the population. One approach is to introduce a parameter p which defines the probability that, at each time step, an agent interacts with the information instead of a peer [González-Avella et al., 2010; Peres and Fontanari, 2010, 2011]. In this case, it was shown that, surprisingly, a large probability to interact with the information actually increases fragmentation instead of favouring agreement. However, this could be explained by the fact that increasing the frequency of interaction to the external agent decreases the possibility of agents to interact between themselves. This, coupled with the fact that, at the beginning, some individuals cannot interact with the information (low overlap), causes an isolation of these and creation of additional clusters. In consequence, methods that try to address this, in the quest for induced agreement, have appeared. For example, [Rodríguez et al., 2009] add a set of "effective features" to the individuals, i.e. additional values in the state vector, that are considered to be always equal to the information (mimicking in this way the way media is designed to target a social group). This causes the overlap with the information to be always non-zero, and leads to better agreement to it. Similarly, in [Mazzitello et al., 2006], a different definition of overlap to the mass media was used, again non-null for all individuals. A different approach can be seen in [Rodríguez and Moreno, 2010]. Here, the so called "social influence" is used, where individuals are affected by all neighbours (including the mass media, with a certain probability defined by its strength), using a procedure similar to voting. Again, this method increases the number of agents adhering to the external information with the increase in the media strength. Several other model extensions have been analysed, trying to combine the effect of media with noise and social network structure [Candia and Mazzitello, 2008; Gandica et al., 2011; Mazzitello et al., 2006; Rodríguez and Moreno, 2010].

In [Quattrociocchi et al., 2011], a Deffuant-like model in two dimensions, with two conflicting opinions (x_i^1,x_i^2), was studied under the effect of external influence from media and experts, on a scale-free social network. The model was applied to opinions on welfare and security. Results showed that when the media message is false, peer interaction can help the population escape the message, only if the media does not reach more than 60% of the individuals.

Chapter 3

Opinion dynamics model with disagreement and modulated information

In the following, we introduce a new modelling approach that allows for multiple opinions on the same issue, and that models the continuous internal state of individuals. The model considers the probabilities that an individual will make a specific choice (such as voting or choosing a market product). The interest is both in agreement and disagreement dynamics. Importantly, an external source of information is included (modelling e.g. mass-media), to allow for an analysis of strength of different information types and effect on the population. The model allows for intrinsically modulated information, a feature which enables important observations about the effects of extreme or mild messages to be made.

Besides the details, which we shall describe in the following section, the model runs in this way. At each time step an individual is chosen randomly and paired with another one; there will be an exchange of information between these two peers and according to how much the two individuals are similar, this information exchange will bring their opinions closer or pull them more apart, so mimicking the dynamics of agreement and disagreement respectively. After this interaction with a random individual, the first chosen individual will interact with the external source of information, so to simulate the influence of mass media. Mass media are considered as an immutable individual.

3.1 Method

A fully connected social network of N individuals is considered, where each agent has to make a choice between K possible opinions on a issue. Each individual maintains a set of probabilities for the possibilities: $x = [p_1, p_2, \dots p_K]$ with

$$\sum_{k=1}^{K} p_k = 1 \tag{3.1}$$

i.e. an element in the simplex in $K-1$ dimensions. Similarity between individuals x^i and x^j is defined as the cosine overlap between the two opinion vectors:

$$o^{ij} = \frac{\sum_{k=1}^{K} x_k^i x_k^j}{\sqrt{\sum_{k=1}^{K} (x_k^i)^2 \sum_{k=1}^{K} (x_k^j)^2}} \tag{3.2}$$

Pairs of individuals (x^i,x^j), randomly selected, interact, either by agreeing or disagreeing, based on their instantaneous overlap:

$$p_{agree}^{ij} = o^{ij} \pm \epsilon \tag{3.3}$$

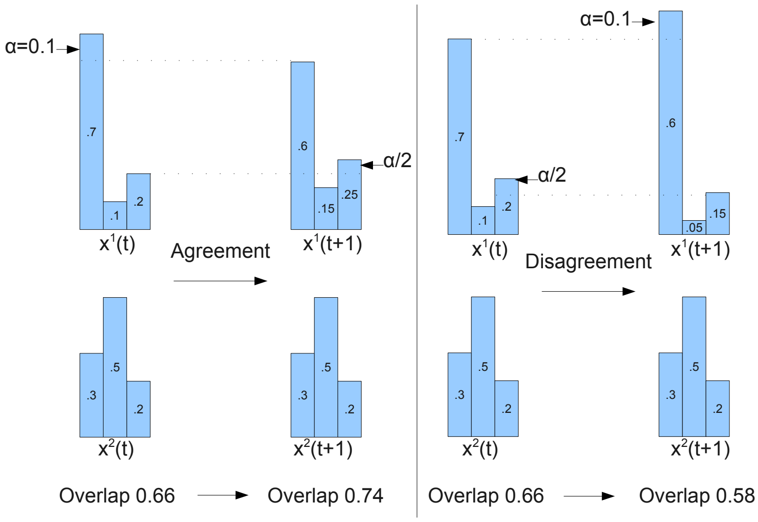

Figure 3.1: Example of pairwise interaction: agreement vs. disagreement. On the left, agreement is demonstrated, where the top individual changes the first opinion value by $\alpha = 0.1$ to become more similar to the bottom individual (and, as such, the overlap increases). On the right, the top individual follows disagreement dynamics, decreasing its overlap with the agent on the bottom. The rest of the opinions values are adjusted by equal amounts ($\frac{\alpha}{2}$) to conserve the unity sum.

$$p^{ij}_{disagree} = 1 - p^{ij}_{agree} \tag{3.4}$$

Here, ϵ is a noise term which avoids lack of interaction due to null overlap. Interaction causes one of the individuals in the pair to change a random position l in the opinion vector:

$$p^i_l(t+1) = \begin{cases} p^i_l(t) \bullet \alpha\, \mathrm{sign}(p^j_l - p^i_l), & \text{if } |p^j_l - p^i_l| > \alpha \\ p^i_l(t) \bullet \frac{1}{2}(p^j_l - p^j_l), & \text{otherwise.} \end{cases} \tag{3.5}$$

where $\bullet$ is $+$ for agreement and $-$ for disagreement. Hence, the position is changed by $\alpha > 0$ a small fixed step, unless differences to the other individual are too small, in which case the change is half the difference. This allows complete agreement between individuals. Parameter α determines the flexibility of agents. The rest of the elements in x^i are adjusted to preserve the unit sum, by redistributing the amount position l was changed by. Figure 3.1 demonstrates graphically the update rule employed.

External information, e.g. mass-media, is introduced as a static agent $I = [I_1, I_2, \ldots I_K]$ with

$$\sum_{k=1}^{K} I_k = 1 \tag{3.6}$$

After interacting with a peer, an individual interacts also with the information with probability p_I, by the same interaction rules. Hence, interacting with the external information does not imply less peer communication. In previous models, external information biased individuals towards one choice out of all possibilities (e.g. Axelrod, Sznajd). Here, this means setting one position in I to 1 and others to 0. In reality, however, sources of information are so wide that only one possibility is never promoted. Our approach has the ability to model such complex influence, by choosing non zero values on more positions of I, i.e. *modulated information*.

The model presented here can be considered an extension of continuous-opinion models (Deffuant, Hegselmann-Krause) to multiple choices. Another extension of Deffuant, with similarities to this approach, was introduced by [Lorenz, 2007], however our model differs significantly from that. Specifically, the similarity measure is defined differently here, and our model does not impose 'bounded confidence', since it is possible that two individuals agree even if their overlap is 0 (with low probability). Furthermore, the model introduced here considers disagreement dynamics and effect of external information, not included in [Lorenz, 2007]. Additionally, unlike Deffuant, agreement is not based on a hard threshold, but is probabilistic, as in the Axelrod model. Similarities to the CODA model [Martins, 2007] can be also observed, since it also includes discrete opinions and continuous internal states. However, in the case of CODA, the analysis is restricted to 2 and 3 opinion choices, and interactions are performed based on the discrete opinion, while in our model agents are aware of the internal state of peers. It is important to note that the model introduced here is different from other extensions of continuous models to vectorial opinions [Deffuant et al., 2012; Fortunato et al., 2005], where each element in the vector relates to a different issue, in the same way as Axelrod. Here, all positions in the opinion vector relates to choices for *the same issue*, such as political vote or choice of phone company.

An important asset of the model presented here is the ability to modulate external information, for multiple choices. This was also true for the Deffuant model: if we consider that the continuous oppinion is actually a probability to make a choice between two discrete options, then milder external information can be introduced by using information values far from the extremes of the opinion interval. In fact, similarities between information effects in the Deffuant model and the one introduced here will be discussed in a later section.

In the following, we are interested in analysing the number of clusters emerging from a random population and in the effect of external information.

3.2 Results

3.2.1 Initial condition

Firstly, an analysis of the influence of the initial condition on the final state will be presented, when no external information is present. A random sampling of the simplex in $K-1$ dimensions yields a population with a relatively large average overlap:

$$\bar{o} = \frac{2\sum_{i,j} o^{ij}}{N(N-1)} \tag{3.7}$$

This value represents the probability that a randomly chosen pair of individuals will follow agreement dynamics, so it may have a large influence on the final state of the population. Thus, the interest is to see how the dynamics depend on this feature of the initial condition. For this, populations with different initial $\bar{o}$ have been generated, by removing from the random sampling some individuals that have entropy ($S = -\sum_{i=1}^{K} p_i \log(p_i)$) larger than a specific threshold. Specifically, individuals with entropy larger than this threshold have a small probability to be maintained in the population. Decreasing the threshold, populations with decreasing $\bar{o}$ can be obtained. Figure 3.2 shows the distribution of populations with different $\bar{o}$ for $K = 3$. Throughout the rest of this paper, more fragmented or compact population will be generated in this manner, with the most compact initial condition corresponding to the random sampling of the simplex.

In order to study the effect of the initial condition, we have performed numerical simulations ($N = 300$, $\epsilon = 0.1$) for different $K \in \{3, 5, 10, 20, 30\}$, with corresponding $\alpha \in \{0.0167, 0.01, 0.005, 0.0025, 0.00167\}$. Hierarchical clustering of the final population has been performed, using complete linkage clustering, and cutting the tree at 0.8 similarity level. This ensures that if two agents i and j are in the same cluster then $o_{ij} > 0.8$. The relative number of

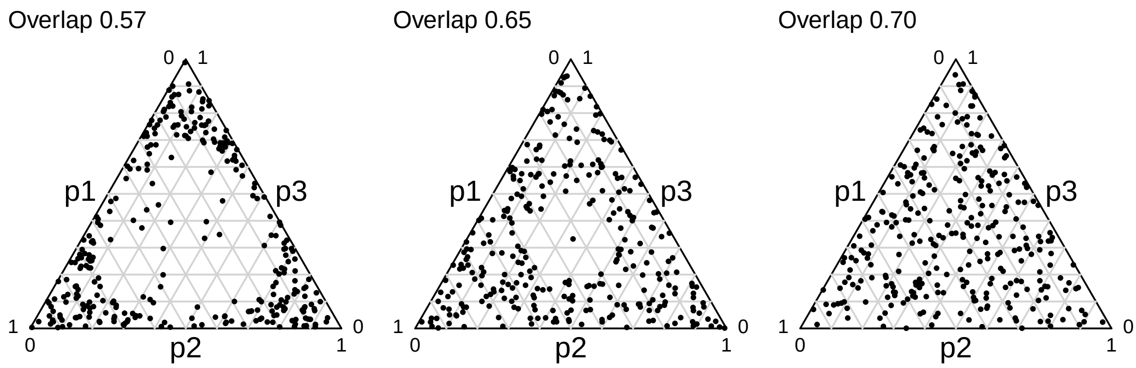

Figure 3.2: Initial conditions for $K = 3$ - distribution of opinion values for a population of 300 individuals, with different average individual overlap.

clusters has been computed as the cluster participation ratio (PR):

$$PR = \frac{\left(\sum_{i=1}^{C} c_i\right)^2}{\sum_{i=1}^{C} c_i^2} \tag{3.8}$$

with C the number of clusters and c_i the size of cluster i. This measure is stronger than the number of clusters itself, as it also considers cluster sizes. For instance, if the population consists of two clusters, PR would be 2 only if the clusters are equal in size, and very close to 1 if one of the clusters is extremely small compared to the other.

Figure 3.3 displays PR values for different realisations of the model, depending on the initial $\bar{o}$. This shows that the initial condition has a large effect on the final population, with a transition in the number of clusters obtained. Specifically, above a certain value of the initial overlap, the population forms one cluster, while below this the population divides into K clusters, each of the form $(1, 0, \ldots)$. This value decreases with K, showing that agreement in the population is facilitated by the existence of more opinion choices. This has also been observed for the model in [Lorenz, 2007], although without preforming an analysis of the initial condition effect. However, it is important to note that in the case of agreement, agents maintain a probability different than 0 for (almost) all options ($p_i > 0$), which means a generalised state of indecision. On the other hand when clusters form, these adopt a more decided option with $p_i = 0$ for many values of i.

3.2.2 Effect of information

To analyse the effect of external information on the final population, numerical simulations have been performed for $N = 300$, $K = 5$, $\alpha = 0.01$, $\epsilon = 0.1$ and p_I ranging from 0 to 1. Four types of information have been employed, in order to study the effect of extreme and mild external messages on the population (specifically $I \in \{[1, 0, 0, 0, 0], [0.8, 0.2, 0, 0, 0], [0.4, 0.2, 0.2, 0.1, 0.1], [0.2, 0.2, 0.2, 0.2, 0.2]\}$. Since, as shown in section 3.2.1, the initial condition plays a very important role in the number of clusters obtained, four different initial conditions have been used, with $\bar{o} \in \{0.49, 0.55, 0.57, 0.62\}$. These values cover situations before, around and after the transition occurs (Figure 3.3), with $\bar{o} = 0.62$ corresponding to the random sampling of the simplex. For each parameter value above, 20 simulations have been performed.

The number of clusters has been determined as in the previous section, using the PR measure. The effect of information has been quantified by computing the average information overlap (IO)

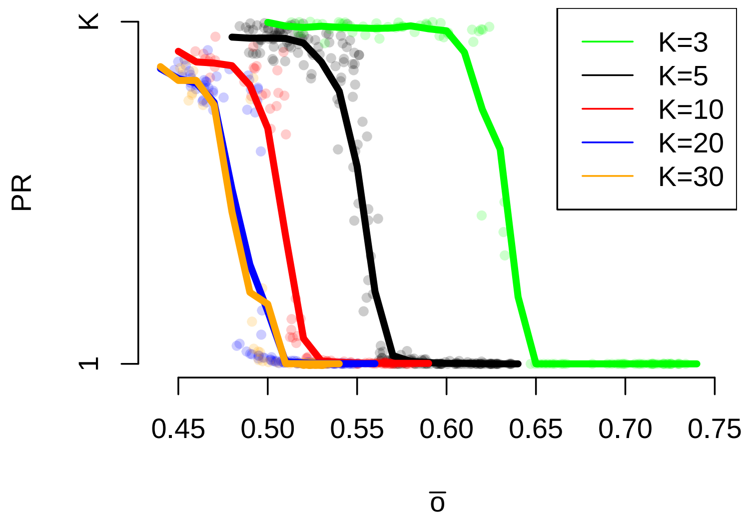

Figure 3.3: Effect of initial condition on the number of clusters in the final population. Dots represent individual instances, while lines are averages.

for the final population:

$$\overline{IO} = \frac{\sum_{i=1}^{N} o^{Ii}}{N} \tag{3.9}$$

where o^{Ii} represents the cosine overlap between agent x^i and the external information I. This average measure is an indicator of the percentage of individuals in the population adhering to the information.

Figure 3.4 displays $\overline{IO}$ and average cluster number for the different parameter configurations and several patterns can be observed. In general, extreme information is less successful in the population compared to milder messages, as $\overline{IO}$ values indicate. Additionally, mild information favours cohesion in the population, i.e. a decreased number of clusters compared to the $p_I = 0$ situation, while extreme information induces segregation (increased PR). These effects increase, in general with p_I. Of course, the extent of the two effects observed depends on the initial condition. That is, the segregation effect of extreme information is small when the population starts from a tight community, i.e. large $\bar{o}$, with the number of clusters increasing when $\bar{o}$ decreases. Similarly, the success of mild information is smaller when the initial population is not very compact and larger in the opposite case.

Several other interesting details can be observed. In general, even when the frequency of interaction with the external information (p_I) is very large, the success in the population (IO) is bounded. This bound depends both on the initial condition and on the type of information, with a smaller value for extreme information and low initial $\bar{o}$, and virtually no bound for mild information and large $\bar{o}$. This is due to disagreement dynamics based on the overlap between individuals and the external information, and is observable also in real life, where no matter how much propaganda there is, if the individuals do not agree enough with an idea, this is not adhered to.

To analyse in more detail the clustering patterns obtained, Figure 3.5 displays histograms of cluster sizes obtained in 20 runs, for $p_I \in \{0, 0.01, 0.5\}$. This shows that, in general, as expected, PR values of 5 correspond to clusters of similar size (around 60 individuals), while PR values close to 1 are generated by one very large cluster and one or several extremely small groups. The figure

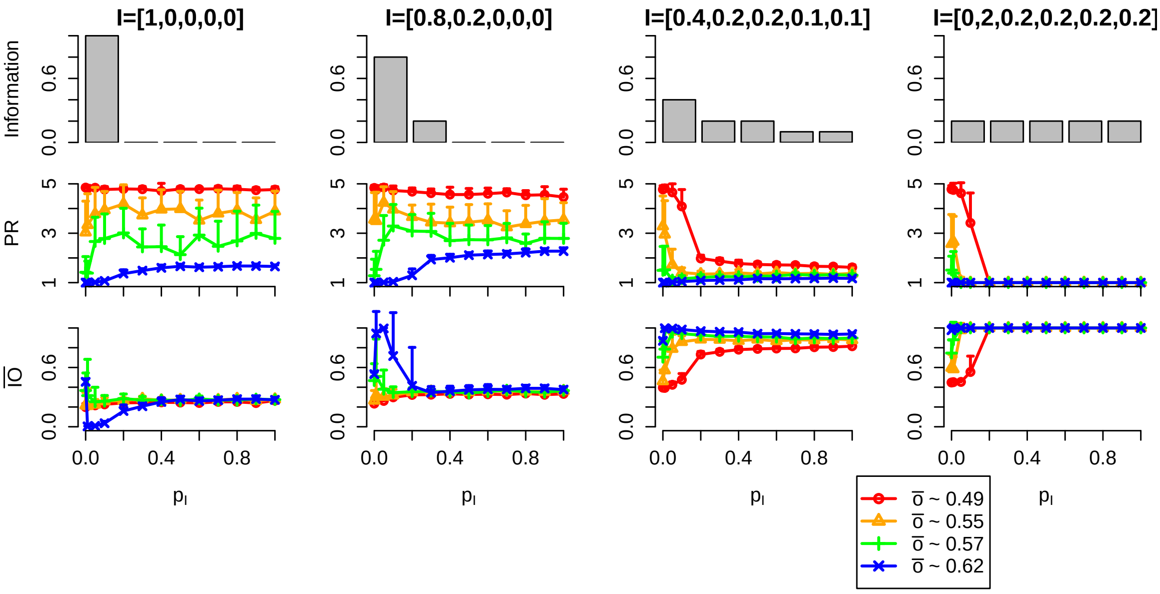

Figure 3.4: Numerical results for a population of 300 individuals, $K = 5$, $\epsilon = 0.1$ and $\alpha = 0.01$ are presented (100000 population updates, 20 instances for each parameter value). Four different information values are used, corresponding to each column. The top graphs show the histogram of the information, the middle the average number of clusters as a function of p_I while the bottom the average information overlap again as a function of p_I. Error bars are displayed as well, showing standard deviation for each point. Where error bars are not visible, the standard deviation is within graphic point limits or null.

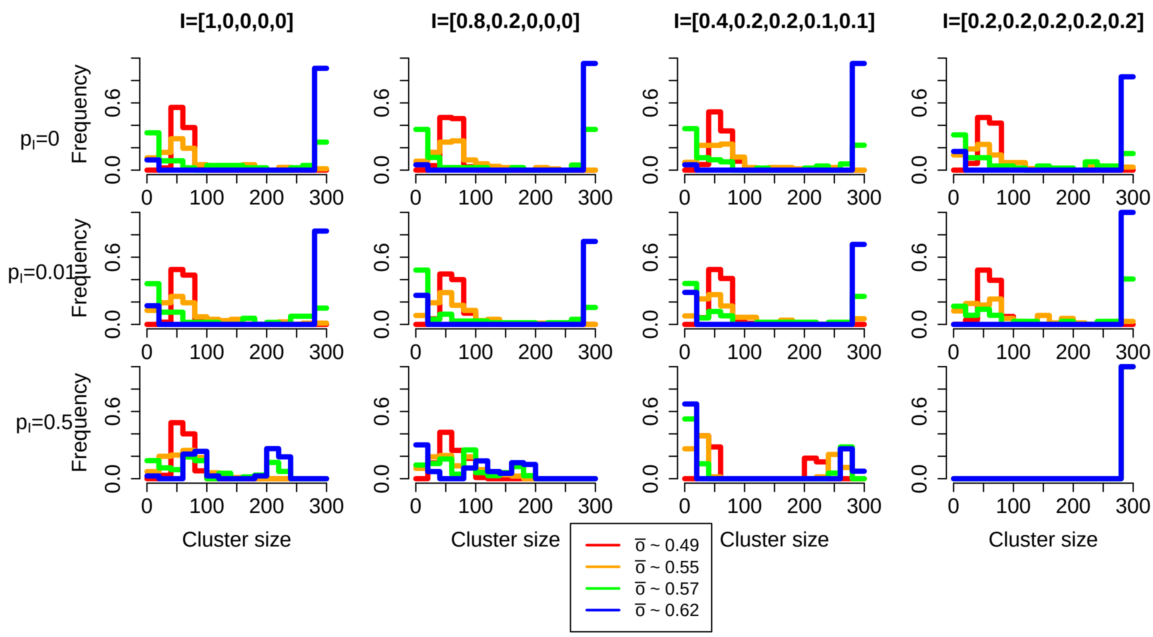

Figure 3.5: Histograms of cluster sizes obtained over 20 simulations runs for $K = 5$, $N = 300$, $\epsilon = 0.1$ and $\alpha = 0.01$. Four information types and four initial conditions are displayed, for $p_I \in \{0, 0.01, 0.5\}$.

shows very clearly how the fraction of very large cluster sizes increases as the initial condition becomes more compact (red to blue lines) and as the information becomes milder (left to right columns).

Another very interesting phenomenon can be observed for compact initial conditions, i.e. random sampling of the simplex (Figures 3.4 and 3.5, blue lines). When the information is not too extreme $I = [0.8, 0.2, 0, 0, 0]$, the entire population adheres to it provided p_I is very small, while as p_I increases, the media success decreases. On the other hand, when the information is very peaked ($I = [1, 0, 0, 0, 0]$), very small p_I leads to complete disagreement to the population, while larger p_I increases agreement. Cluster sizes, however, do not show a large change when increasing p_I from 0 to 0.01, showing that group composition is basically the same, even if IO values change significantly. This indicates that a very low p_I allows for the dynamics to proceed in a similar manner as without information, and after groupings are formed, these are slowly swayed towards or away from the information. In the specific case here, a compact initial population forms first one cluster, which moves close to the information if this is mild, or far from it if peaked. This shows that, when facing a compact group, an external message is more efficient if presented gradually, provided it is not extremely different from the current convictions of the population. This suggests that peer influence is more effective than that from an external static source. This can be explained by the fact that peers are flexible, and move towards others freely, while external information is too rigid. An agent that is facing an external message to which they do not agree, will move away from it, while when the direct exposure to this is small, the peers that are in between this agent and the external information will sway them towards accepting the message. However, for this to happen, the message has to be close enough to the initial state of the population, i.e. acceptable by a large number of individuals. When this is not true, as is the case with an extreme external information, the entire population will disagree with the message, and the media campaign will have no effect.

It is important to note that the overlap of the external message to the population, which, our results show, is one of the most important determinant for the success of a campaign, depends also on K. For instance, an extreme message $[1, 0, \ldots]$ has an average overlap with a random population of 0.447 when $K = 5$ and 0.577 for $K = 3$. Figure 3.6 shows average overlap with the information obtained for different K values. Each initial population has been obtained by random sampling of the simplex, being thus a compact population. Three information types have been used. It is obvious that the three types of information have a very different effect depending on K. While for $K = 3$, information $[0.5, 0.3, 0.2]$ is very mild, having complete success, for $K = 30$ this is actually quite extreme, since only three out of thirty options are promoted, so the information overlap obtained decreases drastically. We can conclude that the success of all three information types in the population decreases with K. This indicates that it is easier to convince the public about a specific option when there are few choices, compared to when the number of choices is large. When p_I is small, the phenomenon explained in the previous paragraph can be observed, for all K, i.e. mild information has complete success, while extreme information fails to attract individuals.

Temporal patterns

For further detail on the dynamics of the system, this section discusses temporal patterns for two initial conditions ($\bar{o} = 0.49$ and $\bar{o} = 0.62$) and two information types ($I = [1, 0, 0, 0, 0]$ and $I = [0.4, 0.2, 0.2, 0.1, 0.1]$), each for $p_I \in \{0, 0.01, 0.5\}$ and $K = 5$. Figures 3.7,3.8,3.9 and 3.10 show single simulation instances for each p_I. Plots display the evolution in time of each of the five elements of the opinion, for every individual in a population of 300. Each row corresponds to one position p_i in the opinion vector, while each column represents a different value of p_I. The value of the information is shown in red. Individual opinions are displayed in colours corresponding to the cluster they belong to at the end of the simulation (e.g. all green lines correspond to individuals

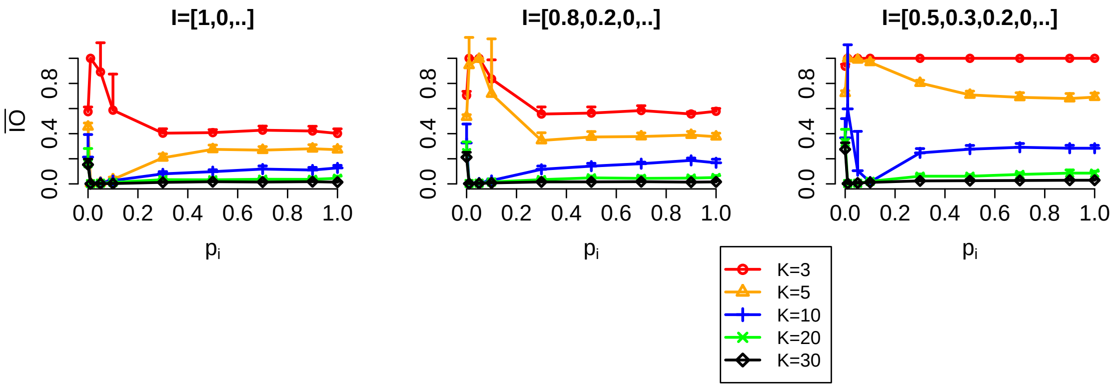

Figure 3.6: Average information overlap for compact populations of 300 individuals, $K \in \{3, 5, 10, 20, 30\}$, $\epsilon = 0.1$ and $\alpha \in \{0.0167, 0.01, 0.005, 0.0025, 0.00167\}$ (10 instances for each parameter value). Error bars show one standard deviation from the plotted mean.

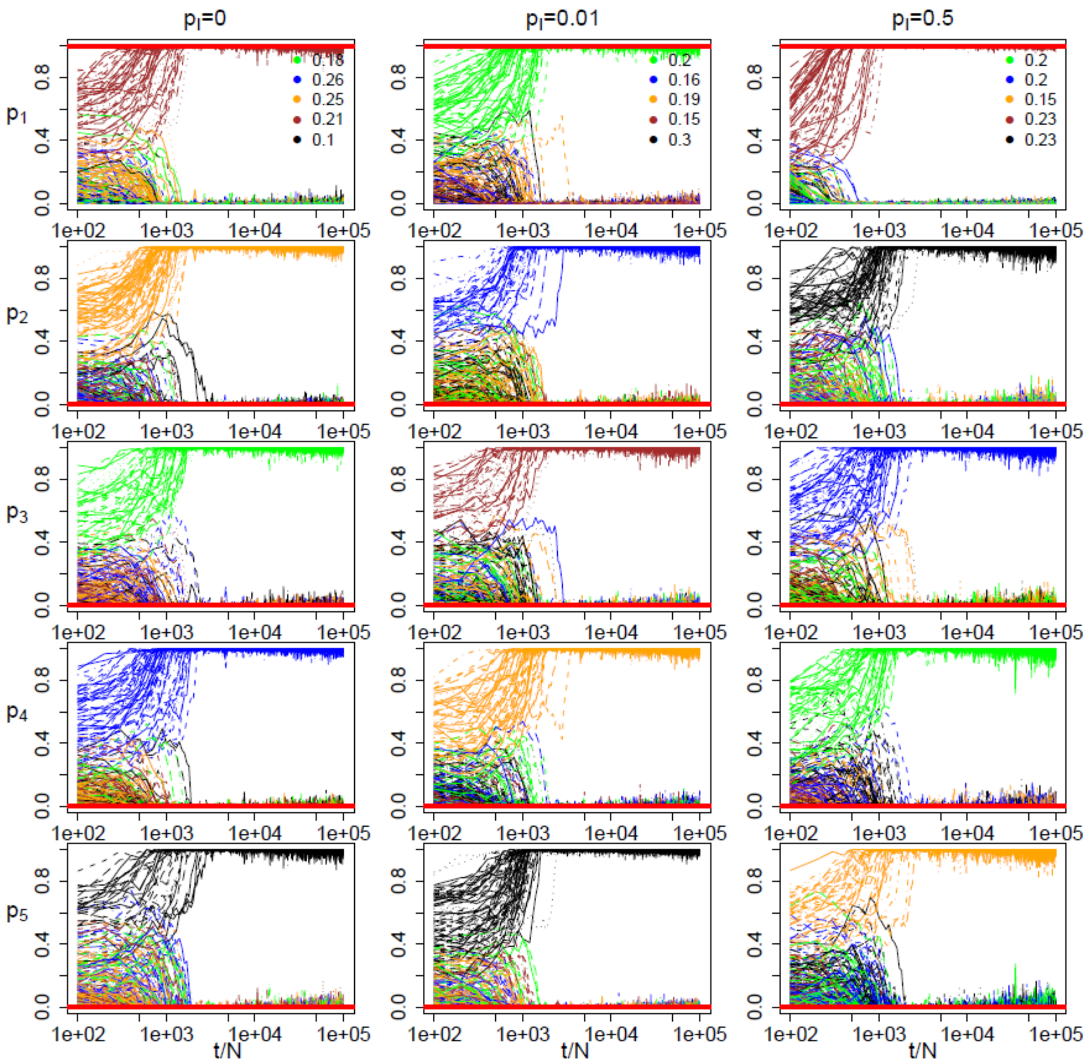

Figure 3.7: Opinion values for $K = 5$, $N = 300$, $\bar{o} = 0.49$, $I = [1, 0, 0, 0, 0]$. Each row corresponds to the positions in the opinion vector $x = [p_1, p_2, p_3, p_4, p_5]$, while each column represents a different p_I. Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red.

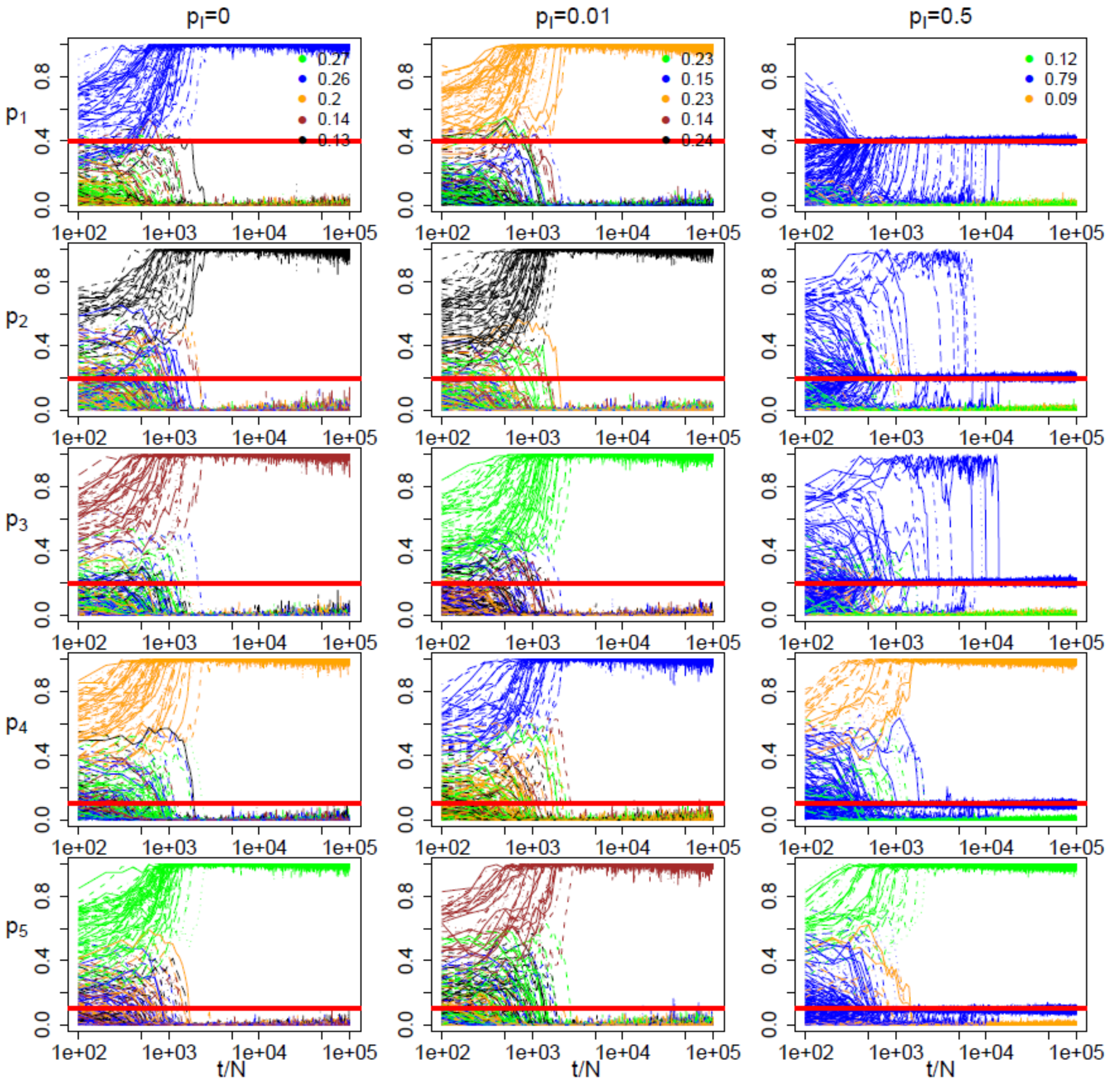

Figure 3.8: Opinion values for $K = 5$, $N = 300$, $\bar{o} = 0.49$, $I = [0.4, 0.2, 0.2, 0.1, 0.1]$. Each row corresponds to the positions in the opinion vector $x = [p_1, p_2, p_3, p_4, p_5]$, while each column represents a different p_I. Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red.

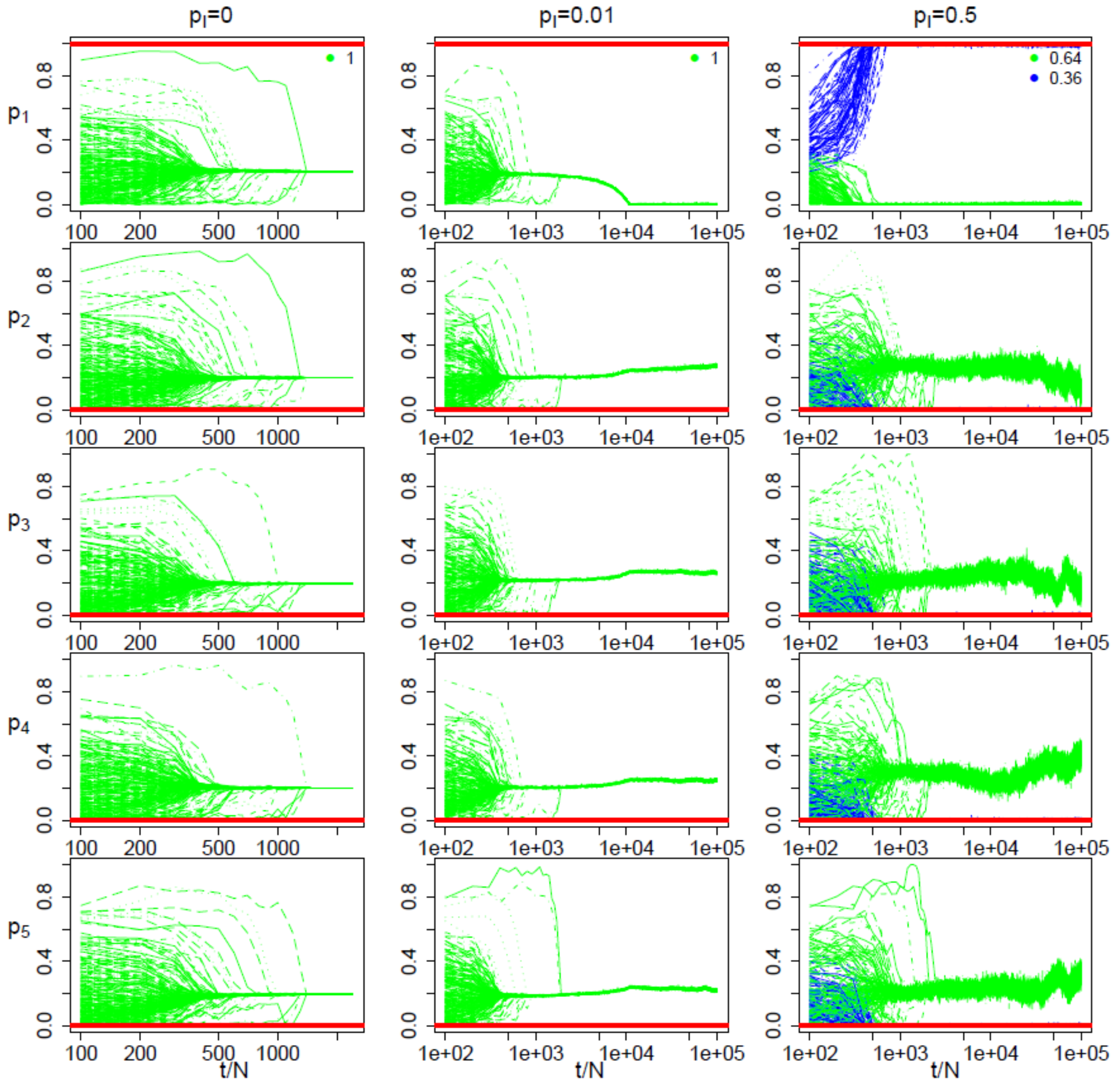

Figure 3.9: Opinion values for $K = 5$, $N = 300$, $\bar{o} = 0.62$, $I = [1, 0, 0, 0, 0]$. Each row corresponds to the positions in the opinion vector $x = [p_1, p_2, p_3, p_4, p_5]$, while each column represents a different p_I. Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red.

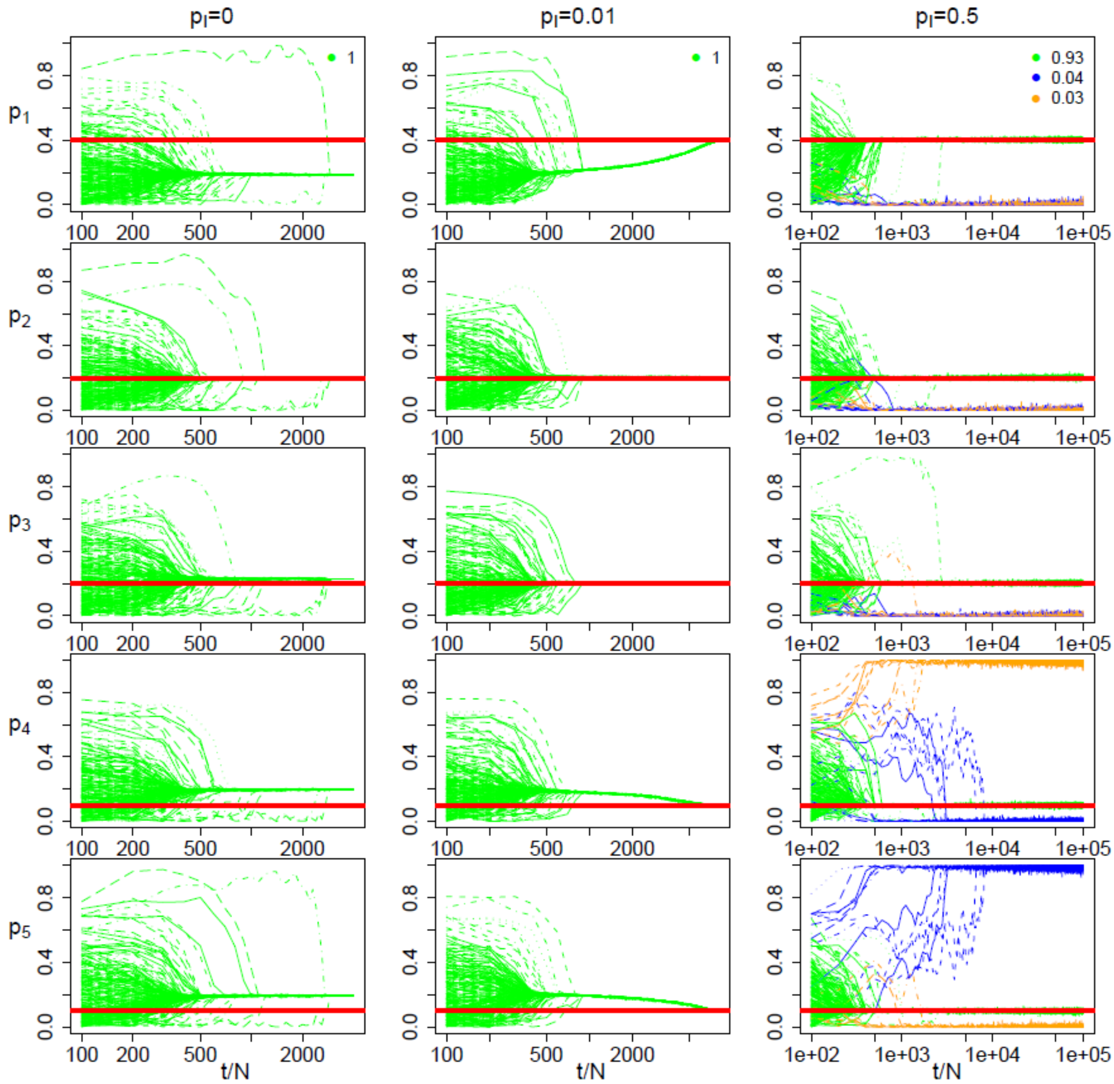

Figure 3.10: Opinion values for $K = 5$, $N = 300$, $\bar{o} = 0.62$, $I = [0.4, 0.2, 0.2, 0.1, 0.1]$. Each row corresponds to the positions in the opinion vector $x = [p_1, p_2, p_3, p_4, p_5]$, while each column represents a different p_I. Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red.

which cluster together). The relative cluster sizes ($\frac{c_i}{N}$) are also shown at the top, as a legend.

As figures show, opinions start in random position spanning the interval $[0, 1]$, and stabilise around a particular value. The system never reaches a frozen state, with small fluctuations preserved even after the clusters are formed. This is due to the parameter ϵ, which allows for agreement even when the overlap between two individuals is 0, or disagreement even when the overlap is one.

For the case of segregated initial populations, Figures 3.7 and 3.8 show the formation of five clusters within the population, for $p_I = 0$. These 5 clusters are maintained for extreme information, regardless of the value of p_I, due to the segregation effect of such information. For milder external message, on the other hand, the five clusters are only maintained when the frequency of exposure is small. In this situation, however, although the average overlap with the information is quite large (Figure 3.4), no individuals agree completely to the information. Larger p_I causes a large cluster to form around the external information value, showing the cohesion effect of a mild message (which appears only when the frequency of exposure is large enough).

Figures 3.9 and 3.10 show similar graphs for a compact initial population. When no information is present, all individuals form one cluster. Plots for $p_I = 0.01$ validate the explanation of the total agreement/disagreement observed in Figures 3.4 and 3.5. Specifically, low exposure to the information allows the population to initially form one cluster, similar to no exposure, which is afterwards slowly affected by the external message. When this is extreme (Figure 3.9), the cluster shifts away from the external information, since it is too dissimilar. On the contrary, when the information is mild, the cluster moves towards it resulting in complete agreement (Figure 3.10). For $p_I = 0.5$, the group, determined previously by the initial condition, does not form, and part of the population adheres to the information directly. For extreme information, the information cluster is small, while for the mild message, this dominates the population.

All in all, we observe that a small p_I allows for dynamics to be determined by the initial condition at the beginning of the system's evolution. Clusters are influenced by information after they are formed: these move away or close to the information value, depending on the cluster overlap with the external input. For larger p_I, the initial overlap of each individual with the external information is important: individuals who are far away form additional clusters that are then too distant from the rest of the population and from the information to be attracted back.

Chapter 4

Preliminary report on emerging awareness

As described in Deliverable 3.1, the Widenoise application allows its users to guess the noise level before the actual measurement with the help of a slider ranging from 0 dB to 120 dB. This user estimate is intended both to introduce a mere playful aspect in the measurement process and to understand whether the act of measuring itself could induce a sort of awareness on the problem of noise pollution. In fact, a comparison between the measured values and the user estimated ones can directly reveal users subjective noise perception and how this changes in time in response to the use of the application. In Fig. 4.1 the guessed noise level vs the measured one is displayed. The size and darkness of circles are related to the expertise of users. Darker and larger circles correspond to samples taken by more experienced users who already collected and guessed measures several times. If a measure is correctly estimated, than the corresponding circle would fall on the bisector line of the picture, i.e. the diagonal of the picture connecting the lower left corner to the upper right one. The figure clearly shows how the guessed values of acoustic levels tend to the measured ones by increasing the application usage, as darker circles thicken toward the figure bisector. Moreover, the off-diagonal scattered circles seem to be biased toward an underestimate of the noise level (tend to be under the diagonal), demonstrating that users are not fully aware of the acoustic pollution around them and improve their awareness by using the application. On the other hand, the light-grey dots on the top of the figure, corresponding to a full-scale estimate of 120 dB, are mostly due to newbie users, who probably did not intend to over-estimate the real measured values, but were rather driven by inexperience and/or laziness. These preliminar outcomes are particularly interesting in the frame of our project, since they demonstrate that users can act as sensors, if properly trained, and can provide a reliable acoustic monitoring of the environment. As a side effect, users acquire awareness on environmental issues. We expect a similar behaviour with the air-pollution application. In Fig. 4.2 the training effect of the Widenoise application on users can be better appreciated with the help of histograms counting the occurrences of the deviation between the guessed noise level and its measured value. The histogram referring to the first measure of users shows a broader shape (top-right), while the subsequent guesses are much closer to the center, i.e. to the measured values.

In the apps, the user is also asked to qualify the noise with four slider about disturbance (“love” vs “hate”), feeling (“calm” vs “hectic”), isolation (“alone” vs “social”) and artificiality (“nature” vs “man-made”). We performed a preliminary analysis of the sliders results for the Rome case study. The sliders were not frequently used so we ruled out the case with all the sliders left at their default values, but we took into account all the other situations (for example, if just one slider was touched we assumed that the others were left in the default position on purpose, and not for laziness, so their default values are significant). The value is a continuous number between 0 and 1. We plotted both the histogram (with bin large 0.1) of the measures and the average level of noise for each bin obtaining the graph in Fig. 4.3. The Widenoise application also allows the annotation of sampled

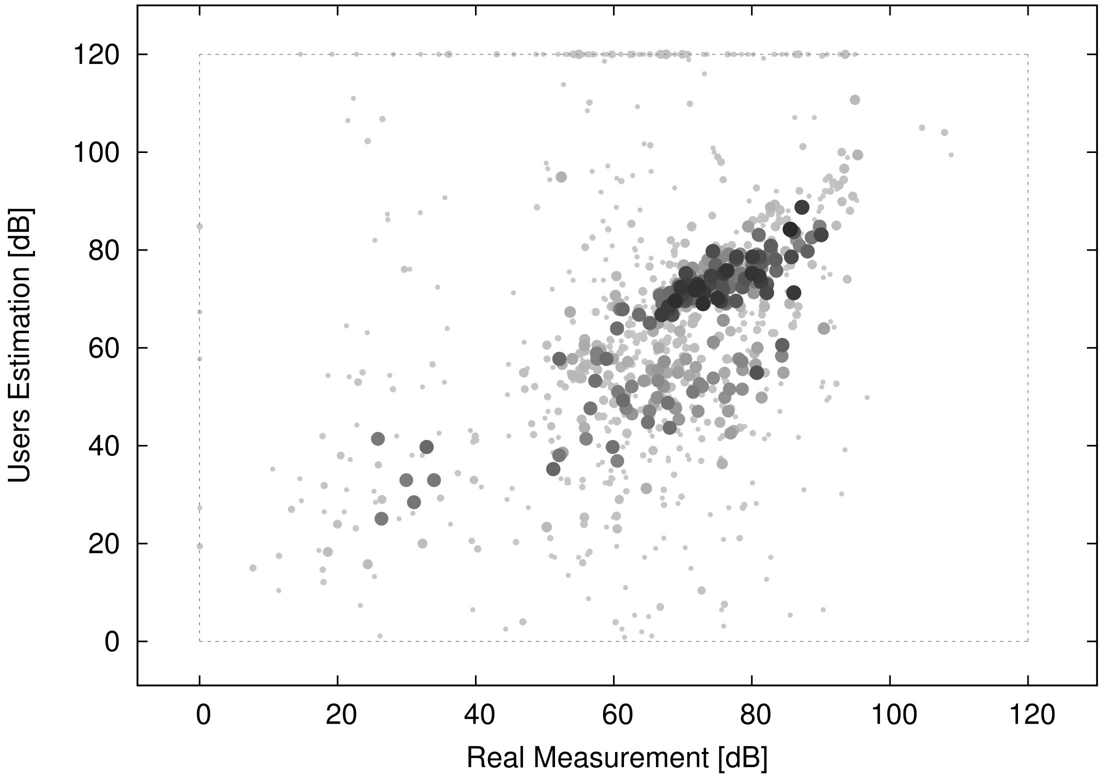

Figure 4.1: Scatter plot of user estimated noise level vs the measured noise level. Circle darkness and size are related to the expertise of users: light-grey smallest circles correspond to the application being used for the first time. A user correctly estimated guess would fall on the picture diagonal connecting the lower left corner to the upper one.

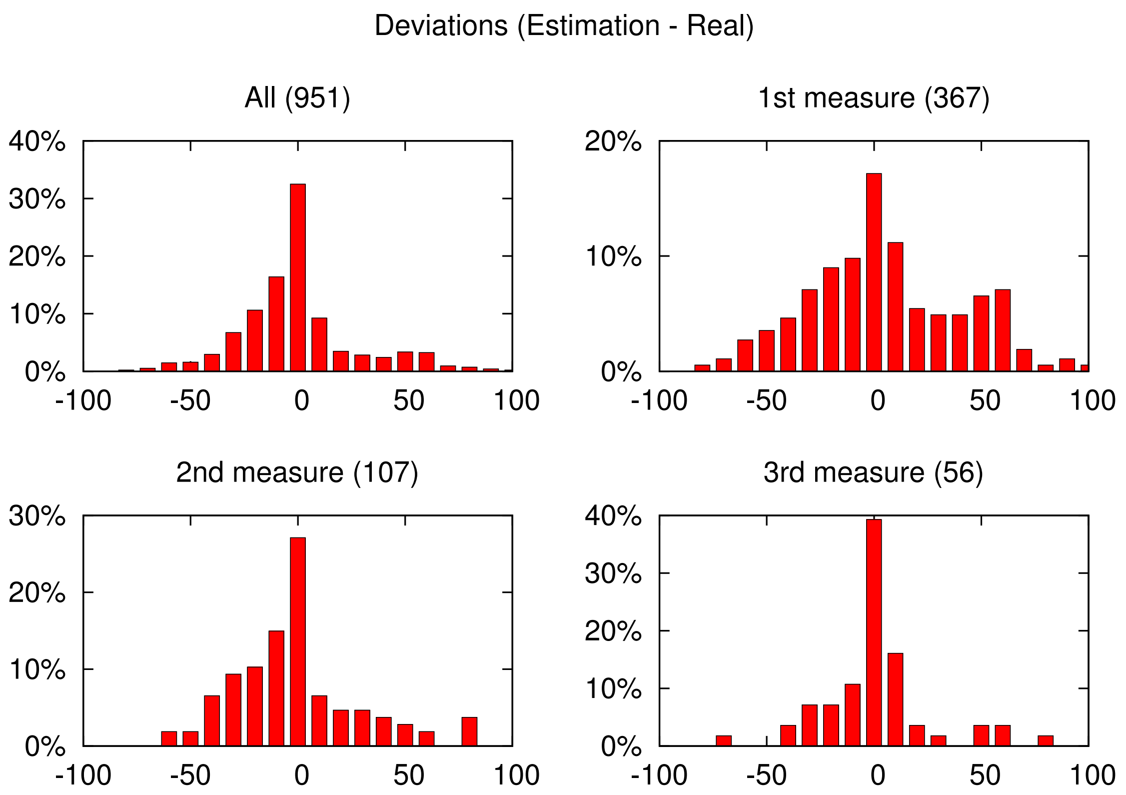

Figure 4.2: Histogram of the deviations between the guessed acoustic level and the measured one. The negative values on the horizontal axis correspon to an underestimated guess. Top-left: overall 951 measures; Top-right: only those 367 measures taken for the first time by users; Bottom-left: only the 107 measures taken for the second time; Bottom-right: the 56 measures taken for the 3rd time. Note how the histogram becomes sharper with increasing application usage. It is also appreciable a remarkable asymmetry of the histograms displaying more underestimated values than overestimated ones (40% vs 27% in the top-left figure).

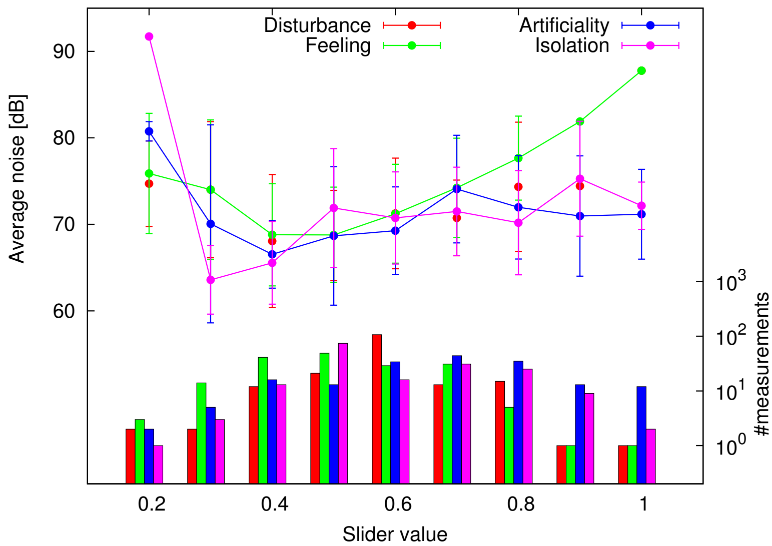

Figure 4.3: The histogram of the sliders measurements and the average with the standard deviation of the noise level of each bin. The slider noise qualifications are disturbance ("love" vs "hate"), feeling ("calm" vs "hectic"), isolation ("alone" vs "social") and artificiality ("nature" vs "man-made").

Figure 4.4: Tag-cloud representing the tags freely inserted by users in Widenoise in order to better qualify the sampled acoustic level. The font size is proportional to the logarithm of the tag frequency.

noise levels with freely chosen tag words. In Fig. 4.4 we show the resulting tag-cloud of this annotation process. The larger tag font size indicates a more frequent usage of the tag. Among all tags we recognize **PLANE**, referring to the Heathrow case study carried on by **UCL**; **ANTWERPEN** and **SAN LORENZO** for the case studies in Antwerp and Rome (details in Deliverable 3.1). The tag **GARDEN** shows that users are also driven by curiosity and use the Widenoise application in less noisy contexts as well.

Chapter 5

Conclusions and Perspectives

An overview of existing methods for modelling opinion dynamics has been presented, with emphasis on the effect of external information and alternative dynamics (e.g. disagreement). As the literature shows, there is a large amount of work performed for discrete opinions (both single-valued and vectorial), such as the voter, majority rule or Axelrod models, while for continuous opinions the effect of information is only studied for single-valued opinions (Deffuant, Hegselmann-Krause), restricting the analysis to only two possible choices. However, it is important to study this effect for multiple choices, as well.

A new model for opinion dynamics was introduced, considering the internal probability of individuals to choose between several discrete options, with both attractive and repulsive dynamics. An important feature of the model is the ability to introduce modulated external information, so that more possibilities can be promoted (e.g. by mass media).

Numerical results showed that extreme information causes fragmentation and has limited success, while moderate information causes cohesion and has a better success in attracting individuals. This coincides to the success of marketing or election campaigns, where coalitions and milder messages are more effective. Additionally, information success is maximised when individuals do not interact too much with the information, showing the importance of the social effect in information spreading. An important factor driving the capacity of information to influence the population appears to be the initial similarity to the agents. This is a known fact in devising marketing strategies, for instance, where information is displayed in a way appealing to the target audience.

Similar effects of external information have been observed previously for other models. For instance, [Gargiulo et al., 2008] observed that aggressive media campaigns are not effective, using the Deffuant model with external information, and that individuals should be exposed to the external influence gradually in order to optimise its success. Further, in [Hegselmann and Krause, 2006; Kurz, 2011], for the Hegselmann-Krause model, it was shown that extreme information causes formation of antagonistic clusters, while mild messages are more successful. A segregation effect from external information has been also observed for the Axelrod model [González-Avella et al., 2010; Peres and Fontanari, 2010, 2011], similar to our observations for extreme information (information in discrete models, such as Axelrod, is equivalent to the extreme information in a continuous model, i.e. promoting one option only).

The initial condition proved to be of large importance in the dynamics, with compact populations resulting in one cluster, while less compact starting points yielding more groupings, with a transition depending on the number of choices present. A similar large effect of the initial condition has been recently shown in [Carro et al., 2012], for the Deffuant model. Additionally, the change in the transition point depending on K, the number of choices, indicated that agreement is easier to obtain for a larger number of choices (with no external information). This is similar to the findings for the model in [Lorenz, 2007], which shares similarities to our approach.

Several further analyses of the model presented are envisioned for the future. These include changing the dynamics to allow individuals to interact on more than one opinion choice and studying different social network topologies (here, all results are presented on a complete network). Additionally, application to real data, in order to simulate observed social processes and be able to make predictions, is required to further validate the model. In the context of the EveryAware project [Everyaware Consortium, 2012], the model will be also applied to simulate behavioural and opinion changes on environmental issues, based on subjective data which will be collected during test cases.

Finally in a preliminary study, we showed how the usage of the Widenoise application to monitor the acoustic level around users, can have an active role in raising users awareness on the noise pollution problem. We found that the sound level guessed by users before taking the sample gets ever closer to the actual measured value with increasing application usage. In this way we train users to act as acoustic sensors themselves.

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