Quantum Electronics Letters

Cavity Length Adjustment and Output FEL Intensity Optimization

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Abstract—**In this letter, we reconsider in more quantitative detail the possibility of enhancing the output intensity of a FEL operating with a bunched electron beam and a finite macropulse duration by adjusting the cavity length while the signal grows.**

As it is well known, the gain of a FEL oscillator, operating with a pulsed e beam, depends on the mismatch $\delta\mathcal{L}$, from the nominal cavity length, necessary to compensate the effect of optical packet velocity reduction, induced by the lethargy and on $\mu_c = N\lambda/\sigma_z$, which measures the relative slippage of the optical packet over the electron bunch (for a discussion on the short pulse effects see [1]). A practical formula, including the above quoted dependences, has been derived (see [2] and references therein) and reads (for the meaning of symbols see Table I)

$$G(\theta, \mu_c) \simeq -0.85 g_0 \frac{\theta}{\theta_s}\left\{\ln\left[\frac{\theta}{\theta_s}\left(1+\frac{\mu_c}{3}\right)\right]-1\right\}$$

$$\theta = \frac{4\delta\mathcal{L}}{g_0\Delta}, \quad \Delta = N\lambda. \tag{1}$$

The quantity θ is a dimensionless parameter and is usually referred to as the cavity detuning parameter.

Its validity has been checked numerically for small and large values of μ_c and analytically for small μ_c only.[a]

The gain relation (1) has been modified in [2] to include saturation effects due to the intensity intracavity growth, namely

$$G(\theta, \mu_c; \bar{I}) = -0.85 g_0 \frac{\theta}{\theta_s}\left\{\ln\left[\frac{\theta}{\theta_s}\left(1+\frac{\mu_c}{3}\right)(1+\bar{I})\right]-1\right\} \tag{2}$$

where $\bar{I}$ is the dimensionless optical intensity ($\bar{I} = I/I_s$, I_s being the FEL saturation intensity which is the quantity halving the small-signal gain). The above expression reproduces (at least qualitatively) two important features

TABLE I

g_0 =	small-signal gain coefficient
$\theta = \frac{4\delta L}{g_0\Delta}$	cavity detuning parameter
$\delta\mathcal{L}$ =	cavity mismatch from the nominal cavity length
$\mu_c = \frac{\Delta}{\sigma_z}$,	$\Delta = N\lambda \equiv$ slippage distance
θ_s =	0.456 synchronous cavity detuning parameter

1) the maximum gain saturates according to a relation analogous to that of conventional lasers

$$G^*(\bar{I}) = \frac{0.85 g_0}{\left(1+\frac{\mu_c}{3}\right)(1+\bar{I})} \tag{3}$$

2) the value of θ yielding the maximum gain decreases with $\bar{I}$ according to

$$\theta^*(\bar{I}) = \frac{\theta_s}{\left(1+\frac{\mu_c}{3}\right)(1+\bar{I})} \tag{4}$$

as a consequence of the fact that with increasing intracavity power gain saturation is induced with a consequent reduction of the lethargy [2].

In [2] it has been pointed out that the output power can be enhanced by moving the cavity mirrors while the intercavity signal grows in such a way that the system is operating at the value of the cavity length [see (4)] providing the maximum gain. Practical solutions to realize the mirror movements were also discussed.

In this letter, we complete the analysis of [2] presenting simple formulas which allow a straightforward evaluation of the above quoted enhancement.

The above relations have been used to follow, after each round-trip, the growth of the dimensionless intracavity power, using the following rate equation

$$\bar{I}_{n+1} = \bar{I}_n + [G(\theta, \mu_c, \bar{I}_n) - \gamma_T]\bar{I}_n \tag{5}$$

where n refers to the round-trip number and γ_T refers to the cavity losses. Imposing the condition

$$G(\theta, \mu_c, \bar{I}^*) = \gamma_T \tag{6}$$

Manuscript received December 3, 1990; revised May 28, 1991.

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IEEE Log Number 9103205.

[a] Using indeed a perturbative procedure (1) has been reproduced up to the second order in μ_c.

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we get the following relation

$$\bar{I}^* = \frac{\theta_s}{\theta\left(1 + \frac{\mu_c}{3}\right)} \exp\left[1 - \frac{\theta_s}{\theta}\frac{\gamma_T}{0.85g_0}\right] - 1 \quad (7)$$

referred as from now on, as "fully saturated intensity" (FSI), exhibiting a maximum at

$$\theta_M = \frac{\theta_s}{0.85} \cdot \gamma_T/g_0 \quad (8)$$

where

$$\bar{I}_M^* = \frac{G^*(0)}{\gamma_T} - 1. \quad (9)$$

The peak output laser intensity is linked to the above quantities by

$$\hat{P}_L = \frac{1}{2N}\chi(\theta, \mu_c)\hat{P}_E \quad (10)$$

where N is the number of undulator periods, $\hat{P}_E$ is the peak e-beam power and

$$\chi(\theta, \mu_c) \sim \frac{\bar{I}^*}{\bar{I}_M^*(\mu_c = 0)}. \quad (11)$$

The comparison of (10) with the predictions of the 1-D numerical analyses, also discussed in [1], has confirmed the validity of the above simple procedure. According to (10) the function $\chi(\theta, \mu_c)$ should be understood as a short pulse correction to the efficiency of a FEL. The FSI is therefore a measure of the maximum attainable output power in a FEL operating in the low gain regime. However, such a value can be however reached only after a number of round-trips and, therefore, owing to the finite duration of the e-beam micropulse, the effectively obtained output power may be lower.

To give an example we have reported, in Fig. 1, $\bar{I}$ versus θ at different round-trips and for $\mu_c = 1$, $\gamma_T = 0.05$, and $g_0 = 0.5$. The FSI (the solid line in the plot) is reached after 300 round-trips. Which amounts, for an optical cavity 6 m long, to a time of about 6 μs.[b] Therefore, the e-beam macropulse should be at least 6 μs long. We must also underline that the FSI, for different θ, is reached at different times. In fact the region corresponding to that around the maximum gain is reached in a relatively shorter time (in about 50 round-trips in the case of Fig. 1). This fact is, perhaps, better clarified in Fig. 2 where we have reported $\bar{I}$ versus n (i.e., the round-trip number) at different values of θ. Furthermore, in Figs. 3 and 4 we show the same as in Fig. 2 for different parameters; it is evident that the time to reach the FSI, is larger with smaller gain.

In [2] we have raised the question whether it is more convenient to operate with a fixed cavity length (around θ_M) or move the mirrors (while the signal grows) from $\theta^*(0)$ to θ_M, satisfying at any time during the interaction, the maximum gain condition (4). We have concluded that the second solution is by far more convenient for a larger g_0/γ_T ratio. In this letter we dwell on this point trying to clarifying it in a more detailed and quantitative way.

We must preliminarily emphasize that the small-signal

[b]The time to reach the FSI should not be confused with the rise time, which in the case is much smaller (2.24 μs).

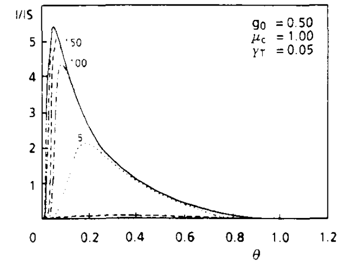

Fig. 1. Normalized intensity $\bar{I}$ versus θ, at different round-trips for $g_0 = 0.5$, $\gamma_T = 0.05$, and $\mu_c = 1$.

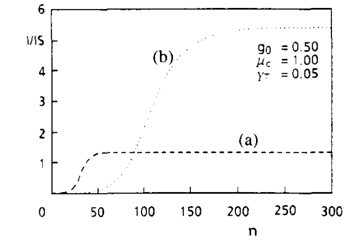

Fig. 2. $\bar{I}$ versus n at different θ (n denotes the number of round-trips). (a) $\theta = \theta_s/(1 + \mu_c/3)$ and (b) $\theta = (\theta_s/0.85)\gamma_T/g_0$.

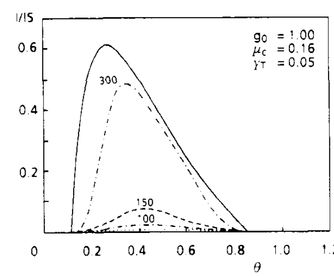

Fig. 3. $\bar{I}$ versus θ at different n for $g_0 = 0.1$, $\gamma_T = 0.05$, $\mu_c = 0.16$.

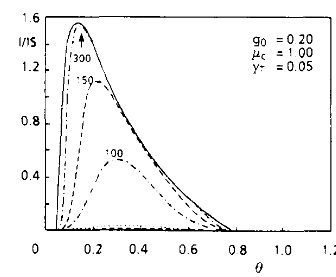

Fig. 4. $\bar{I}$ versus θ at different n for $g_0 = 0.2$, $\gamma_T = 0.05$, $\mu_c = 1$.

net gain at θ_M is not zero but [inserting (8) into (1)]

$$G(\theta_M, \mu_c) = -\gamma_T \ln \left(\frac{\gamma_T}{G^*(0)}\right) \tag{12a}$$

or equivalently [see (5)]

$$G(\theta_M, \mu_c) = \gamma_T \ln (1 + \bar{I}_M^*). \tag{12b}$$

The above net gain is small, we expect therefore that, if we set the cavity at θ_M, the system will eventually saturate at I^* but in a longer time. There are three possible options

1) to work with fixed cavity at $\theta^*(0)$ [see (4), i.e., maximum small-signal gain]

2) to work at θ_M

3) to work moving θ, from $\theta^*(0)$ to θ_M according to the previous prescription (following $\theta^*(\bar{I})$ while $\bar{I}$ grows).

The intensity growth per round-trip is shown in Figs. 5 and 6, for different values of the main parameters. In case 1, saturation levels lower than $\bar{I}_M^*$ are reached; on the other side, both configurations 2) and 3), eventually reach the FSI, but the last case yields this value in a time which is shorter with increasing g_0/γ_T ratio. We must underline that the mirror movement solution seems to be advantageous in any case, since it allows the buildup of the radiation from the very beginning, together with a large output signal. Before concluding this letter, let us add information on the orders-of-magnitude of the times and displacements involved in the mirror movements. According to (4) and (8) we find that the required θ variation is given by

$$\Delta\theta = \theta_M \left[\frac{G^*(0)}{\gamma_T} - 1\right] \tag{13}$$

and therefore the cavity should be displaced by the following quantity

$$\Delta \mathcal{L} = 1.3 \times 10^{-1} [G^*(0) - \gamma_T] \,\Delta. \tag{14}$$

The square brackets in the above equation contain the small-signal net gain. Unfortunately, there is not an analogous simple expression for the time, or the number of round-trips, in which the displacement, given by (10), should be achieved. Just to give some numerical examples, we find that a FEL operating at 1 μm with $N = 50$ and a cavity 12 m long, requires $\Delta \mathcal{L} = 2$ μm in $\Delta t = 0.8$ μs (namely the mirrors should be moved with an average velocity of 2.5 ms^{-1}).

A further point to be stressed is that the curve (c) in Figs. 5–7 has been obtained moving the mirrors not continuously, but following the signal growth according to (4). The θ displacement velocity (called characteristic velocity v_c) can be obtained differentiating (4) with respect to the round-trip number and thus getting

$$\frac{d}{dn}\theta(\bar{I}) = -\frac{\bar{I}}{1 + \bar{I}} G^*(\bar{I}) [G^*(\bar{I}) - \gamma_T]. \tag{15}$$

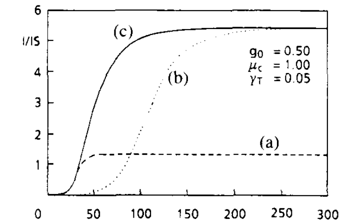

Fig. 5. $\bar{I}$ versus n (a) and (b) as in Fig. 3, (c) with movable mirrors ($g_0 = 0.5$, $\gamma_T = 0.01$, $\mu_c = 1$).

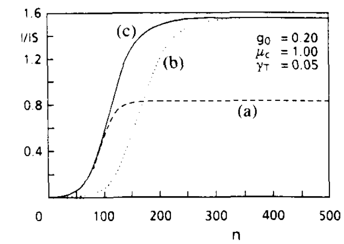

Fig. 6. $\bar{I}$ versus n (a) and (b) as in Fig. 3, (c) with movable mirrors ($g_0 = 0.2$, $\gamma_T = 0.05$, $\mu_c = 1$).

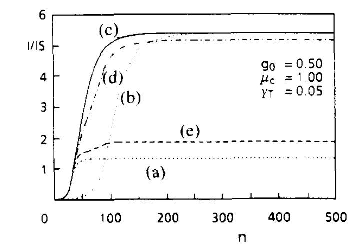

Fig. 7. $\bar{I}$ versus n (a), (b), and (c) as in Figs. 6 and 7. (d) movable mirrors with velocity $\bar{v}_c$ (see text); (e) movable mirrors with velocity $\frac{1}{4}\bar{v}_c$. It must be understood that in the cases (d) and (e) when the plateau is reached, the mirrors are not moved any more.

It is difficult in practical cases, to follow exactly the above velocity "law," e.g., it is easier to move the mirrors at a constant speed. The modification in the round-trip optical power growth are shown in Fig. 7, in which the curve (d) has been obtained shifting the mirrors with a constant velocity, corresponding to the average characteristic velocity $\bar{v}_c$. Finally, the curve (e) has been obtained moving the mirrors with a constant speed of about $\frac{1}{4}\bar{v}_c$.

References

[1] W. B. Colson and A. Renieri, *J. de Physique*, vol. C1, 1983.

[2] G. Dattoli, A. De Angelis, G. P. Gallerano, and A. Torre, submitted for publication.