Gain saturation in bunched free-electron lasers

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We propose a model of gain saturation in free-electron lasers operating with a bunched electron beam. A simple method to evaluate the steady-state intensity as a function of the optical cavity length is given and the result is in close agreement with the intensity measured in the Los Alamos free-electron-laser experiment [B. E. Newnam *et al.*, Nucl. Instrum. Methods A **237**, 187 (1985)].

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Simple models, based on reliable physical assumptions, have allowed a deeper understanding of the basic free-electron-laser (FEL) mechanisms. This is indeed the case of the spectral bandwidth in FEL oscillators, which has been treated by Kim within the context of an appropriate physical ansatz and with a relatively simple mathematics [1]. As far as FEL oscillators operating with a nonrecirculated electron beam are concerned, the results of Ref. [1] rely upon the assumption that the FEL gain exhibits a dependence versus the laser intensity analogous to that of conventional lasers [2]. Such an hypothesis, put forward in Ref. [3], has been checked and confirmed by a careful numerical analysis. In this paper we will show that the gain-saturation scaling can be extended to the FEL operating in the pulsed regime, thus including lethargy and longitudinal-mode-competition effects. We will also show that the analytical results, for example, for the output intensity, are in close agreement with the experimental data.

Denoting with I_S the saturation intensity and with $g=0.85g_0$ the small-signal gain (see Table I for further specification concerning the symbols), the scaling relation proposed in the second citation of Ref. [3] reads

$$G(\bar{I})=g\frac{1-\exp(-\bar{I})}{\bar{I}}\ ,\quad \bar{I}=\frac{\pi}{2}\frac{I}{I_S}\ . \tag{1}$$

The above formula was justified by drawing a parallel between the FEL and conventional laser dynamics. The physical role of the FEL saturation intensity can be easily understood. In conventional lasers I_S is the intensity that halves the population inversion. In the free-electron laser, the dynamics is governed by the pendulum equation

$$\ddot{\Psi}=-\Omega^2\sin\Psi\ , \tag{2}$$

where Ψ, defined as

$$\Psi=(k+2\pi/\lambda_u)z-\omega t-\phi_l\ , \tag{3}$$

is the relative phase between the electron and the electric field, and

$$\Omega^2=\frac{e^2B_0E_l}{(m_0c\gamma)^2} \tag{4}$$

is the frequency of the slow-motion oscillation, which is proportional to the electric field E_l. It can be shown that for $I=I_S$ (see Table I) we have $\Omega L/c\cong\pi$, which means that at the end of the undulator, the electrons trapped in closed orbits have traveled halfway around in the phase-space plane.

TABLE I. Specification of symbols.

$g_0=\frac{4\pi}{\gamma}\frac{I_e}{I_0}\frac{\lambda L_u}{\Sigma_E}\left[\frac{\Delta\omega}{\omega}\right]_0^{-2}F(\xi)$
$I_S=\frac{1}{8\pi}\left[\frac{\gamma}{N}\right]^4\frac{1}{\{\lambda_u K[J_0(\xi)-J_1(\xi)]\}^2}\frac{m_0c^3}{r_0}$, saturation intensity
$I_0=\frac{ec}{r_0}\equiv$ Alfvén current (1.7×10^4 Å)
$I_e\equiv$ Electron-beam current
$\left[\frac{\Delta\omega}{\omega}\right]_0=\frac{1}{2N}\equiv$ homogeneous bandwidth
$N\equiv$ number of undulator periods
$\lambda_\mu\equiv$ undulator period
$L_u=N\lambda_u\equiv$ undulator length
$T_{\rm rf}\equiv$ radio frequency period
$F(\xi)=\xi[J_0(\xi)-J_1(\xi)]^2$, $\xi=\frac{1}{2}\frac{k^2}{1+k^2}$
$\lambda_0=\frac{\lambda_u}{2\gamma^2}(1+K^2)\equiv$ laser wavelength
$\gamma=\frac{E}{m_0c^2}$
$K=\frac{eB_0/\sqrt{2}\lambda_u}{2\pi m_0c^2}\equiv$ undulator parameter
$\theta=\frac{4\delta L}{g_0\Delta}$
$\theta_s=0.456$
$L_{c0}=cT_{\rm rf}$ cavity length
$\delta L=$ displacement from L_{c0}
$\Delta=N\lambda_0\equiv$ slippage length

One of the distinguishing features of the FEL in the bunched regime is the so-called lethargic behavior [4]. If a FEL is driven by the electron beam produced by a rf accelerator, with a bunch-to-bunch distance given by cT_{rf}, one of the constraints necessary for the FEL operation is that the light pulses and the electron bunches overlap in the interaction region. This is achieved by imposing the condition

$$L_c = T_{\mathrm{rf}} \frac{c}{n} , \tag{5}$$

where L_c is the cavity length, and n is an average refractive index found by the light during one round trip. The light interacts, with positive gain, with the electrons traveling at a speed lower than c. The refractive index n is then larger than 1 and depends on the strength of the interaction, i.e., from the gain itself. The FEL gain is then a function of the cavity length and of the slippage distance, which is the difference between the distance covered by the light and by the electrons while passing through the undulator

$$\Delta = L_u(1-\beta_z) = N\lambda_0 . \tag{6}$$

A quantitative analytical picture of the dependence of the gain versus the above parameters is given by the supermode (SM) theory [5]. In Ref. [5] it has been shown that the gain (of the first supermode) can be parametrized in terms of the cavity shortening δL from the empty value ($n=1$) and in terms of the slippage distance, as

$$G(\theta,\mu_c) = -g\frac{\theta}{\theta_s}\left\{\ln\left[\frac{\theta}{\theta_s}\left[1+\frac{\mu_c}{3}\right]\right]-1\right\} , \tag{7}$$

where

$$\theta = \frac{4\delta L}{g_0 \Delta} \tag{8}$$

and

$$\mu_c = \frac{\Delta}{\sigma_z} , \tag{9}$$

with σ_z being the electron-beam rms length. The value $\theta_s = 0.456$ is relevant to the synchronism cavity length for the quasicontinuous electron-beam operation ($\mu_c \to 0$). The above relation is valid in the small-signal limit and takes into account the cavity shortening due to the lethargic effect. When the laser intensity grows, the gain is reduced because of the saturation process. As a direct consequence, the lethargic effect becomes less important and the optimum cavity length in saturated conditions is generally different from that in the small-signal regime. The behavior is analytically reproduced assuming the following gain function, which should extend the validity of (1) and (6) to the bunched electron-beam operation and to the saturated regime, respectively:

$$G(\theta,\mu_c,\bar{I}) = -g\frac{\theta}{\theta_s}\left\{\ln\left[\frac{\theta}{\theta_s}\left[1+\frac{\mu_c}{3}\right]f(\bar{I})\right]-1\right\} , \tag{10}$$

where

$$f(\bar{I}) = \frac{\bar{I}}{1-\exp(-\bar{I})} . \tag{11}$$

We have, indeed, that the optimum cavity-length correction is a decreasing function of the laser intensity

$$\theta_{\max} = \frac{\theta_s}{[1+(\mu_c/3)]}\,\frac{1-\exp(-\bar{I})}{\bar{I}} \tag{12}$$

and that the maximum gain (in θ) scales with the intensity according to Eq. (1).

The behavior of the laser intensity in steady-state conditions can be obtained by imposing that the net gain over one round trip is zero, i.e.,

$$G(\theta,\mu_c,\bar{I}) = \frac{\Gamma_t}{1-\Gamma_t} , \tag{13}$$

where Γ_t are the overall round-trip losses.

Once the values of θ and μ_c are given, the above equation implicitly specifies the steady-state intensity, whose value can be obtained with the help of a pocket calculator. Figure 1 shows the plot of the steady-state intensity versus the parameter θ. The values of the other parameters, i.e., the saturation intensity, the μ_c, and the losses, are the same as those of the Los Alamos FEL experiment, reported in Ref. [6]. The squares refer to the experimental data reported in the quoted reference. The agreement between the experimental results and the results obtained with Eq. (13) is apparent. The data have been compared imposing that the maxima are coincident, since there is no information about the absolute cavity length of the experiment. The different behavior of the tail of the theoretical curve is probably due to the difference between the Gaussian longitudinal electron-beam distribution assumed in the SM theory and the Lorentzian shape reported in the experiment. Moreover, Eq. (13) has been obtained assuming the contribution of the first SM only. Especially in small μ_c conditions, when the small-signal gain is nearly the same for all the

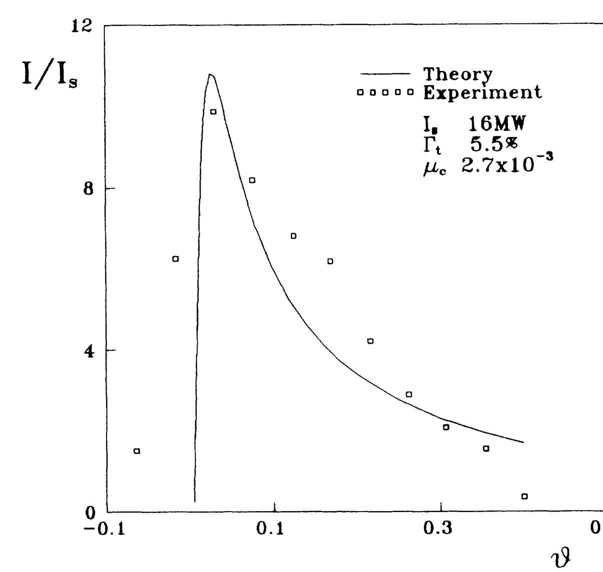

FIG. 1. Steady-state intensity vs θ.

SM's (see, e.g., Ref. [3]), Eq. (13) is affected by an error due to the contribution of all the higher-order SM's that should reduce the sharp decrease of the intensity when the cavity length approaches the nominal value. We must stress that the result summarized in Fig. 1 is essentially analytical since no fit parameters have been included in Eq. (10).

In Ref. [3] other types of gain-saturation formulas have been proposed,

$$G(I/I_s)=\frac{g}{1+I/I_S} \tag{14a}$$

and

$$G(I/I_S)=\frac{g}{1+(1-\alpha)I/I_S+\alpha(I/I_S)^2}\ , \tag{14b}$$

where α has been fixed around 0.14 by fitting the numerical data. Equations (14) have the advantage of being very close to the usual expression given for conventional laser saturation. Equation (14a) holds, however, for the not strongly saturated regime, while (14b) holds even for $I/I_S \gg 1$. The drawback of (14b) is the extraneous parameter α, which cannot be accounted for on an analytical basis. To appreciate better the difference between the various gain versus intensity scaling relations, we plot in Fig. 2 a comparison between Eqs. (1), (14a), and (14b) and the numerical solution. The extension of the above formulas to the bunched-beam regime is still given by Eq. (10), simply redefining the function $f(\bar{I})$ in Eq. (11) as

$$f(I/I_S)=\frac{g}{G(I/I_S)}\ , \tag{15}$$

where $G(I/I_S)$ is given by Eqs. (14a) and (14b). Even if the agreement with the numerical data is slightly worse and their range of application is in some way restricted, Eqs. (14a) and (14b) have the advantage of providing an explicit expression for the steady-state intensity, which reads

$$I_{st}/I_S=-\beta\ , \tag{16a}$$

$$I_{st}/I_S=\frac{\alpha-1}{2\alpha}\left[1-\left[1-\frac{4\alpha}{(\alpha-1)^2}\beta\right]^{1/2}\right]\ , \tag{16b}$$

where

$$\beta=1-\frac{\theta_s}{\theta}\frac{1}{1+\mu_c/3}\exp\left[1-\frac{1}{g}\frac{\Gamma_t}{1-\Gamma_t}\frac{\theta_s}{\theta}\right]\ . \tag{17}$$

In Figs. 3 and 4 we plot the steady-state intensity

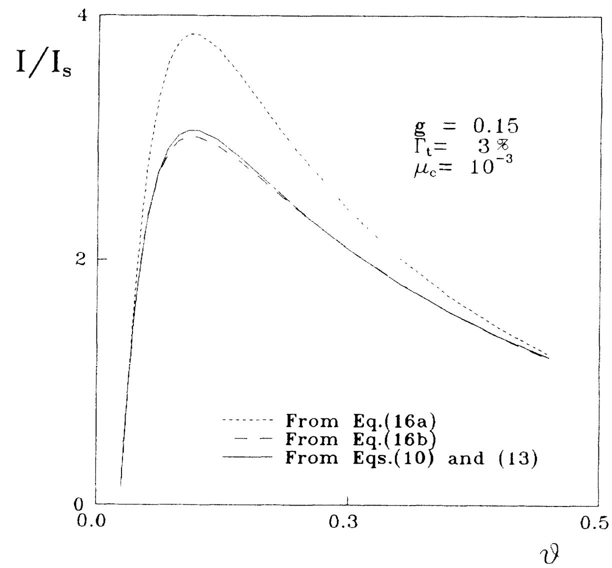

FIG. 3. Steady-state intensity vs θ calculated from different gain vs intensity scaling relations, considering a small-signal gain $g=0.15$.

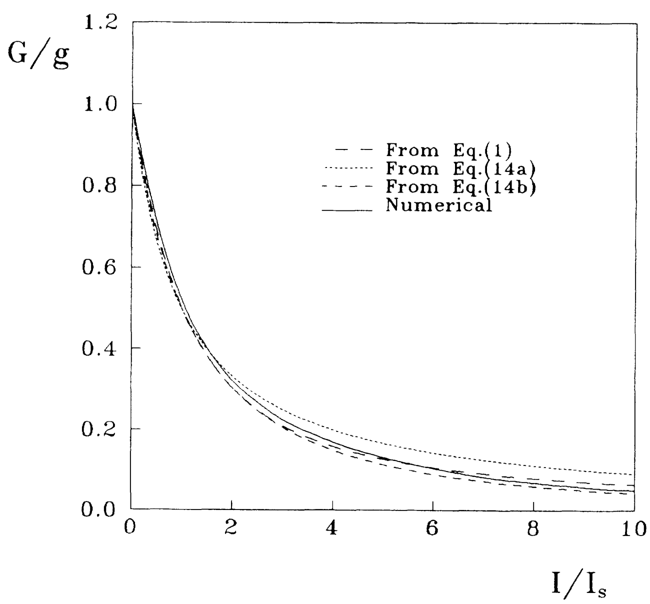

FIG. 2. Gain vs intracavity intensity for different scaling relations.

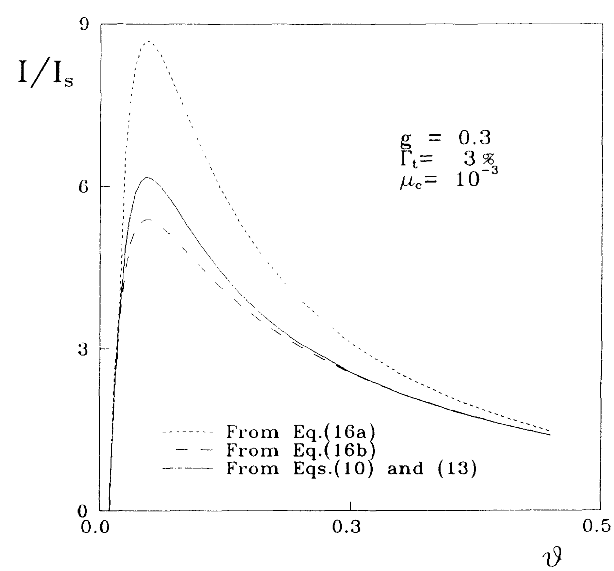

FIG. 4. Steady-state intensity vs θ calculated from different gain vs intensity scaling relations, considering a small-signal gain $g=0.3$.

versus θ obtained from Eqs. (13), (16a), and (16b). It is evident that when the ratio Γ_t/g_0 becomes large, the prediction of (16a), based on the scaling relation (14a), becomes less reliable.

The above expressions show that simple models for FEL saturation can be developed. The equations we proposed indeed yield a good agreement with both experimental and numerical results. We believe, therefore, that this type of analysis can be an efficient tool to obtain preliminary information on the FEL saturated behavior. Furthermore, combining the analysis developed so far with that of Ref. [1], other important information such as the bandwidth dependence on the cavity mismatch can be obtained.

[1] K. J. Kim, Phys. Rev. Lett. **66**, 2746 (1991). For further insight on the problem of FEL linewidth, see also A. Gover, A. Amir, and L. R. Elias, Phys. Rev. A **35**, 164 (1987); I. Kimel and L. Elias, *ibid.* **35**, 3818 (1987); **38**, 2889 (1988); B. Levush and T. M. Antonsen, Nucl. Instrum. Methods Phys. Res. Sect. A **285**, 136 (1989); T. M. Antonsen and B. Levush, Phys. Fluids B **2**, 2971 (1990).

[2] W. W. Rigrod, IEEE J. Quantum Electron. **QE-14**, 377 (1978).

[3] G. Dattoli, S. Cabrini, and L. Giannessi, IEEE J. Quantum Electron. (to be published); Phys. Rev. A **44**, 8433 (1991).

[4] W. B. Colson, in *Laser Handbook,* edited by W. B. Colson, C. Pellegrini, and A. Renieri (North-Holland, Amsterdam, 1990), Vol. 6.

[5] G. Dattoli and A. Renieri, in *Laser Handbook,* edited by M. L. Stitch and M. S. Bass (North-Holland, Amsterdam, 1985), Vol. 4.

[6] B. E. Newnam, R. W. Warren, R. L. Sheffield, J. C. Goldstain, and C. H. Brau, Nucl. Instrum. Methods A **237**, 187 (1985).