

Bubbling and Large-Scale Structures in Avalanche Dynamics

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Using a simple lattice model for granular media, we present a scenario of self-organization that we term self-organized structuring where the steady state has several unusual features: (1) large-scale spatial and/or temporal inhomogeneities and (2) the occurrence of a nontrivial peaked distribution of large events which propagate like “bubbles” and have a well-defined frequency of occurrence. We discuss the applicability of such a scenario for other models introduced in the framework of self-organized criticality.

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One of the major challenges of statistical physics in recent years has been to address the question of the ubiquity of fractality and scale invariance phenomena in nature [1]. The occurrence of such scaling properties in an extremely broad (and unrelated) class of problems calls for a concept of extreme generality, beyond the level of proposing a model suited for any specific phenomenon. A major step in this direction was taken by Bak, Tang, and Wiesenfeld [2] who proposed the notion of self-organized critical phenomena (SOC).

The philosophy of SOC is to note that a precise dynamics can be associated with most second order critical phenomena. This corresponds generically to an infinitesimal external forcing of the system which renders the critical point an attractor of the dynamics. Qualitative predictions of this scenario are the appearance of power-law distributions of avalanches, $1/f$ noise, and more generally the absence of characteristic length or time scales in the dynamics. Examples range from sandpiles [2] and earthquakes [3] to the speciation of living organisms on Earth [4].

The purpose of the present Letter is to report on some unusual features of self-organization occurring during the restructuring of a granular medium [5] under internal avalanches [6,7]. In particular, we show that the system spontaneously breaks spatial homogeneity to develop large-scale structures which trigger (almost) periodic emission of large avalanches that propagate like solitary bubbles. As a consequence, the avalanche distribution exhibits a nontrivial bump for large avalanche sizes. This mechanism can coexist with small scale scaling in some cases while controlling it in other cases. We term this phenomenon self-organized structuring. We show that this mechanism sheds a new light on phenomena usually interpreted in the framework of SOC, such as the celebrated Burridge-Knopoff model [8] for solid friction and earthquakes [9]. It also calls attention to finite size effects, which may alter the proper understanding of the large scale physics at play. The self-organized structuring presented here may also help to understand the often debated question of the relevance of the original model of

SOC, the sandpile model, to real avalanches in granular media as observed experimentally [10].

We illustrate self-organized structuring using a model for describing the restructuring of a granular medium caused by an infinitesimal perturbation (a quasistatic flow of particles out of a vessel). The description of the granular medium is based on the “Tetris” model [11], a toy model designed to account for geometric frustration. This model has been shown to reproduce a number of complex features observed experimentally [12,13] in granular matter, such as the slow dynamics of compaction under vibration [11], segregation [14], etc.

In the simplest version of the model, particles are represented by rectangles of uniform size $a \times b$ which are distributed on the sites of a square lattice with only two possible orientations (length of the particle aligned along one of the principal axes). The size of the rectangle (in mesh units) is chosen so that $a > 0.5$ and $a + b < 1$ (all results quoted below are, however, independent of the exact values of a and b). Geometrical frustration results from the constraint that two particles should not overlap. Figure 1 shows a local arrangement of the particles.

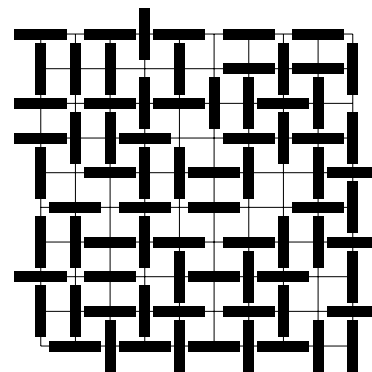


FIG. 1. Tetris model: Sketch of one possible local arrangement of particles in the Tetris model without gravity. The sites of a square lattice can host elongated particles shown as rectangles. The width and length of the particles induce geometrical frustration.

We now consider a system of size $L_x \times L_y$ with periodic boundary conditions along the horizontal direction (parallel to x) and the principal axis of the square lattice pointing along the directions $(1, \pm 1)$ (tilted square lattice). Gravity is along the $-y$ direction. This system is filled with Tetris particles by random deposition under gravity; i.e., particles having one or the other orientation are dropped from a random position at the top. They then fall downwards to any of the two nearest neighbor sites available and so on, executing a directed random walk till they reach a site from which they cannot fall any farther due to excluded volume constraints. This defines the initial state of the system.

The system is then progressively emptied from the bottom row by removing one single particle at a time. Once a particle is removed, other particles may fall down, and induce what we call an “internal avalanche,” i.e., a restructuring of the medium in the bulk to recover a new stable configuration. A new particle is then deposited under gravity at the top. This process conserves the number of particles in the system, and after a large number of avalanches, a steady state is established (in which we make all our measurements). The number of particles which move after the removing of a single grain is by definition the avalanche size.

From the definition of the model, it appears to be a good candidate for exhibiting self-organized criticality. A minimal flow is forced through the system, since the system is required to relax to a stable configuration before a new particle is removed from the bottom line. If the system is very dense, the progressive removal of particles from the bottom line will decompactify it. If it is too loose, then avalanches will propagate easily to the top surface and hence will rapidly increase the density. This competition of two effects can be expected to lead the system to states such that large avalanches have a vanishing but nonzero probability to occur.

Related models have been proposed in the past and it has been claimed that the avalanche size distribution is power-law distributed [7]. Indeed, such a statistics recorded over moderate systems gives rise to a distribution which can be reasonably fitted by a power law as shown in Fig. 2 (see Ref. [6] for a more detailed discussion).

The avalanche size distribution exhibits the occurrence of a well defined bump at the maximum avalanche size (as also observed in other models). This bump could simply represent a finite-size effect due to a looser surface packing. However, we find that the bump is much bigger than what is expected from just this argument and it is not possible to collapse curves obtained with different system sizes satisfactorily with the usual finite-size scaling (the shape of the bump changes as the system size changes, thus disallowing a satisfactory collapse). This is deeply related to the space-time inhomogeneities self-generated by the dynamics. Extensive numerical simulations reveal that the bump displays the following characteristic features: it

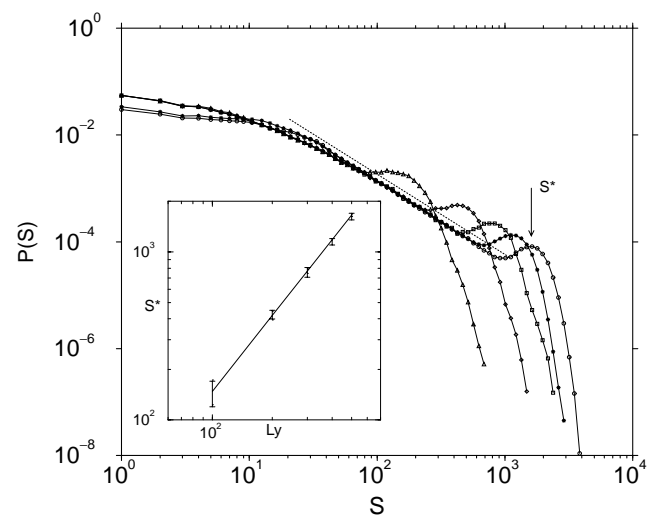


FIG. 2. Avalanche distribution: avalanche size distribution obtained in systems of size $L_x = 100$, and $L_y = 100, 200, 300, 400, 500$. The distribution shows a scaling region characterized by an exponent $\tau \approx 1.5$ and a well-defined bump for large avalanches. The dotted line is a power-law fit with $\tau \approx 1.5$. The top of the bump scales with the height of the system as $S^* \sim L_y^{1.5}$ (see inset).

corresponds to avalanches of a special type whose typical size s^* scales only with the height of the system and is independent of its width.

$$s^* \approx AL_y^\alpha. \quad (1)$$

We found $\alpha = 1.5$ as shown in the inset of Fig. 2.

Since the size of these avalanches is well defined, we could identify them easily in a time series, and thus study the distribution of time intervals, T , between such avalanches. We find that the distribution of T is peaked signifying the occurrence of big avalanches at a well-defined frequency. This also implies a screening effect inhibiting the occurrence of large avalanches close to each other. Therefore, the system displays memory effects over large time intervals which is one of the features of what we call self-structuring.

The rules of the model are time independent, and thus if a memory effect emerges, it has to be encoded in the structure of the medium as spatial inhomogeneities. The peculiar scaling of the large avalanches [Eq. (1)] also points to the same. In order to study this point more quantitatively, we studied the time average of the local density at every single site (i.e., the average probability of occupation of a small region centered around the site). Figure 3 shows a plot of this time-average density map on a system of size $L_x = 100$ and $L_y = 200$ (averaged over 10^5 time steps). We observe clearly on this map alternating channels of low and high density regions which extend from the bottom to the top of the system. These channels have a fixed width w independent of L_x (provided the latter is large enough) and scaling with L_y as $L_y^{0.5}$ [15]. It is important

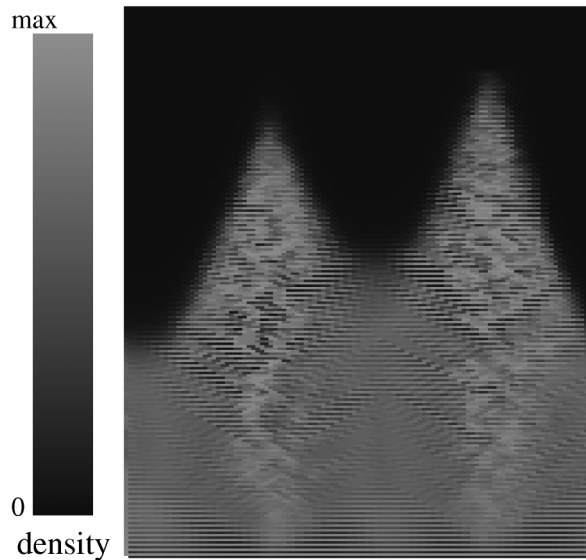


FIG. 3. Density map: Plot of the time-averaged density at every site of the lattice. Alternating low (darker) and high (brighter) density regions are clearly visible. Most large avalanches occur in the low-density channels.

to note that these channels were not present initially but have been progressively carved out by the repeated passages of avalanches. A qualitative interpretation of this phenomenon is that avalanches propagate more easily in less dense regions, but in doing so, they make the density even lower (note, however, that the density cannot become lower than the least stable density of the packing). Thus an instability mechanism results by means of which regions of higher density become progressively quenched in the medium, remaining stable themselves while transferring the disturbance immediately to a nearby low density channel.

These channels become fuzzier close to the bottom line because the continued removal of particles at the bottom forces the flux to be uniform along this line. It is also important to note that the range of variation of the density is quite moderate (order of 10^{-2}), and hence it is necessary to perform a very long time average to be able to capture this effect. A snapshot of the system at a single time does not reveal these channels. This spatial organization is the second hallmark of self-organized structuring.

We are now in a position to study the spatial and temporal structure of the large avalanches. We tailored a system of width $L_x = 40$ and $L_y = 200$ so that only a single channel would appear. We repeated the above procedure of time averaging the density of particles, but now *during* an avalanche, when it has reached a given height. The resulting maps are shown in Fig. 4. We now see clearly that a large avalanche consists of a “bubble” of low density which propagates along the previously shown channel. This bubble is initially rather diffuse but becomes mature at intermediate heights, and preserves its shape and size like a solitary wave, as it propagates upwards to the free surface.

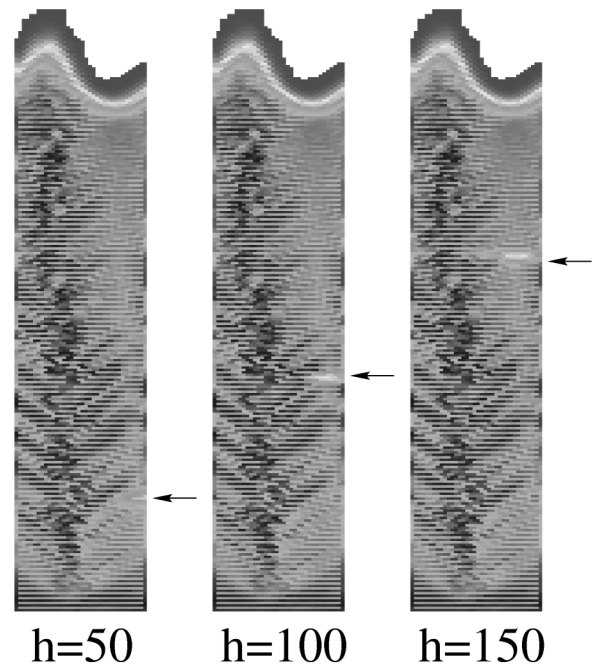


FIG. 4. Bubbles: Emergent spatial structure in a time average performed over the avalanches while they are crossing a certain height h marked in the figure. The front (marked by the arrows) propagates upwards as a localized low density (appears lighter) region (the “bubble”) preserving its size and shape.

These results show that large avalanches consist of bubbles propagating in low density channels at regular time intervals. The latter is just the time needed to nucleate such a bubble by the accumulative effect of small avalanches which deposit voids in the medium. This nucleation stage is the one where (apparently power-law distributed) small avalanches are observed. Further the scaling exponent shown in Eq. (1) can be explained considering the fact that large avalanches are constrained by the size of the low-density channels which have a width vs height scaling as mentioned above.

We have demonstrated that the peculiar statistics of avalanches in the model discussed above is due to a large-scale spatial organization and long time memory which naturally emerge from complex interactions between elements. This model is, however, just one example of such a phenomenon (in the same way as the sandpile model of Ref. [2] was only an example of self-organized criticality). We now claim that a similar scenario may also be at play in other models which have often been discussed in the context of self-organized criticality.

One famous example is the Burridge-Knopoff model [8], a one-dimensional spring-block model for friction introduced in order to understand the dynamics of earthquakes. The model consists of a one-dimensional chain of blocks connected by springs and driven through additional soft springs connected to a rigid rod moving at a vanishingly small velocity. The friction law which governs the interaction between the blocks and their support is a

velocity weakening law. This model was studied in detail by Carlson and Langer [3] who reported that the statistics of slip events was power-law distributed, a property reminiscent of the observed Gutenberg-Richter law in earthquake statistics [9], and of self-organized criticality. The most remarkable aspect of this model is that no noise is explicitly introduced, and a nonuniform motion is only due to an intrinsic instability of the velocity weakening friction law.

In spite of the fact that this point did not raise much attention, the statistical distribution of the length of slip events displays an initial part (reasonably described by a power-law distribution) along with a significant bump at sizes equal to the system size. In fact, in terms of energy released during a slip, it is the latter which contribute the most (which is somewhat in conflict with geophysical data). A detailed numerical investigation [16] allowed one of us to get some information about the structure of these large slip events. They can be shown to consist of localized pulses propagating through the system and occurring at a well-defined time interval. Driving the system at a nonvanishing speed [17] gives rise to similar pulses occurring at fixed time intervals: an organized state where noise disappears. This point indeed shows that the time invariance of this model is broken, and thus that the model belongs to the class of self-organized structuring models.

After the initial proposal of Bak *et al.* [2] who exemplified the notion of SOC by a sandpile model, numerous attempts have been made to measure power law distribution of avalanches in real granular media. It was soon realized that for large system sizes, the statistics of avalanches was no longer a power law but consisted rather of a peaked distribution of large avalanches, plus a tail for smaller ones. This form was interpreted as resulting from an hysteresis in the repose angle of granular media rather than a single critical slope as would be predicted by a SOC scenario. We can understand this effect from the point of view of self-structuring, as the consequence of a slow nucleation effect of avalanches. From simple conservation laws this induces a hysteresis in the angle of repose.

Needless to say, our objective is not to deny the relevance and importance of self-organized criticality. On the contrary, the number of examples where genuine SOC has been demonstrated (e.g., the Abelian sandpile model [18]) speak for themselves. The point we wish to stress is that in some cases, large scale instabilities may modulate the response of the system at a fixed frequency, giving rise to a statistics of events or avalanches which is not purely scale invariant but rather displays a characteristic size (which depends on the structures spontaneously arising in the steady state and thus not trivially related to the system size), a feature we termed "self-organized structuring." Moreover, the breaking down of the translational time invariance of

the system can be accompanied by a corresponding spatial structure which emerges from the dynamics of the system. In contrast, usual SOC is expected to be observed in similar conditions for stable systems, where no long-range structures survive, either in space or time. We also note that the stability or instability of these large structures might be difficult to analyze *a priori*, and thus establishing the self-organized critical or self-organized structuring character of the system may require large scale numerical studies of these models, in order to distinguish these structures from the large amplitude small-scale background noise inherent to these models.

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