Memory in aged granular media

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Abstract. – Stimulated by recent experimental results, we simulate "temperature"-cycling experiments in a model for the compaction of granular media. We report on the existence of two types of memory effects: short-term dependence on the history of the sample, and long-term memory for highly compact (aged) systems. A natural interpretation of these results is provided by the analysis of the density heterogeneities.

The study of the rheology of granular media represents nowadays an important chapter in the more general context of the so-called soft condensed matter [1]. The general questions one is interested in concern the response behaviour of a generic granular medium submitted to an external perturbation. In this spirit it has been recognized that the response of such systems depends in a non-trivial way on the mechanical properties of the grains composing the medium, on the boundary conditions, on the driving procedure (the way to inject energy to perturb the system) and, last but not the least, on the past history of the system, *e.g.* on the procedures the system has undergone until the moment we study it.

In this context the similarities between granular media and the phenomenology of glassy systems have been exploited. One of the main results has been to recognize that in a compaction procedure, where a certain system increases its density if shaken or tapped, a granular medium displays ageing [2] and its properties, *e.g.* the two-times correlation function, depend on the time where one performs the measurement [3, 4], *i.e.* on the age of the system.

The analogy with glassy systems has mainly allowed to export tools and techniques from one field to the other. Typical experiments on glassy systems consist in monitoring the effect of changes imposed to the external temperature on the macroscopic observables [5]. Translated in the framework of granular matter, *i.e.* of a non-thermal system, this means to analyse the effect of changes in the driving procedure, *e.g.* in the tapping amplitude Γ, which, in a typical compaction experiment, represents the control parameter.

In a previous work [4], we have presented a detailed study, in the framework of a microscopic model for granular media, focusing on the response to small perturbations in the driving rate. We have pointed out the importance of the interplay between the response properties and the spatial structures that spontaneously emerge as a consequence of the dynamics

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imposed to the system. In particular we have studied the relevance of the density hetero-geneities in the process of granular compaction, and shown that the density is not the only relevant information about the system. Configurations with the same density but reached with different dynamical procedures show completely different rheological properties. This was nicely confirmed by a set of recent experiments [6]: three systems prepared at the same density but in different ways display different behaviours if the same tapping acceleration is applied to them. This feature concerns the coding of the system history in the microscopic configurations.

In the same set of experiments, similar in spirit to the temperature-cycling experiments in spin-glasses [5], the authors of [6] have pointed out the existence of memory effects in the compaction of granular media. In particular, they performed the study of the effect of relatively large and abrupt changes in the tapping acceleration. This is an efficient way to obtain reliable experimental data since the effect of small changes in the tapping acceleration is difficult to detect in real experiments. What was shown in [6] is that, during the compaction, the response to an abrupt change in the tapping acceleration Γ is opposite (at least at short times) to what could be expected from the long-time behaviour of the compaction: while at constant Γ a higher compaction rate is obtained for larger Γ, a sudden increase in Γ leads to a decompaction, and a sudden decrease gives rise to a temporary increase in the compaction rate. This behaviour is however transient, and the usual compaction rate is recovered after a while: the authors therefore call this effect "short-term memory". The same qualitative behaviour has been obtained also in experiments of compaction under shear [7].

In order to push forward this kind of studies we have extended our work [4] to study the response to large changes in driving. In particular, we explore the behaviour of the Tetris Model [8,9] for abrupt changes in the driving rates in various time regimes, that were not considered in the preliminary, time-limited, experiments [6]. We also explore the return to the initial tapping acceleration and the presence or absence of memory, in a way similar to the temperature-cycling experiments in spin-glasses. We perform these temperature-cycling experiments in non-equilibrium (ageing) as well as in time-translation invariant situations. We hope in this way to stimulate new and more precise experiments along the same lines.

The model we consider, the so-called Tetris Model [8], consists of particles on a tilted (two-dimensional) square lattice, which has lateral periodic boundary conditions ([1]) and a closed boundary at the bottom. In the original model [8], the particles are elongated and pointing in the directions of the lattice; they cannot overlap, which provides very strong constraints and frustration (for example, two identical particles, *i.e.* pointing in the same direction, cannot occupy two neighbouring sites in this direction). Gravity and shaking are implemented as follows: the particles diffuse on the lattice, with the limitations induced by the geometrical constraints, and probability p_{up} to move upwards (with $0 < p_{\mathrm{up}} < 1$), and probability $p_{\mathrm{down}} = 1 - p_{\mathrm{up}}$ to move downwards. The quantity $x = p_{\mathrm{up}}/p_{\mathrm{down}}$ can be related to the adimensional acceleration Γ used in compaction experiments [10] through the relation $\Gamma \simeq 1/\log(1/\sqrt{x})$.

With respect to its original definition we will consider a random version where each particle can be schematized in general as a cross with 4 arms (in general the number of arms is equal to the coordination number of the lattice) of different lengths (instead of a rod), denoted by $l_{\mathrm{NE}}, l_{\mathrm{NW}}, l_{\mathrm{SE}}, l_{\mathrm{SW}}$, chosen in a random way (see [4,9] for details).

The sizes (width × height) used ranged from 200×60 to $20, 40, 60 \times 200$, in order to test various aspect ratios. Though most of the simulations were realized with 60×60 systems, we did not observe any qualitative differences using different aspect ratios. The interesting

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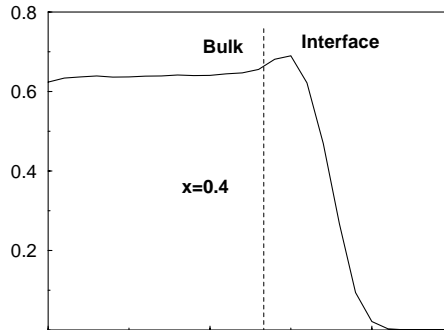([1])The use of closed lateral boundary conditions does not change the results.

Fig. 1 – Typical density profile obtained after $t = 10^4$ Monte Carlo steps at constant forcing $x = 0.4$.

question about the existence of scaling relations for the relaxations as a function of the aspect ratio is beyond the scope of this study.

The procedure used in the experiments and in our simulations is inspired by classic experiments in spin-glasses. The system is initialized by random addition of particles, and is made to evolve with a constant external forcing x_1 up to a certain time t_w. Two copies of the system are then made: one copy is kept evolving with x_1 at all times for reference, while the second copy evolves with a different forcing x_2 for a time Δt, after which the forcing is set back to x_1. We monitor the mean height, the bulk density (defined as the average density in the lower 50% of the system), and the density profiles of both copies during the whole evolution for various values of the difference $x_1 - x_2$, t_w and Δt. We recall that the density profiles are defined averaging the density along the horizontal direction, giving thus indications on the heterogeneities along the vertical direction (see fig. 1), and allowing to roughly distinguish two regions, the bulk and the interface. Before analyzing the different regimes, let us recall a few general features of the compaction at constant forcing. At short times, the density is an increasing function of the forcing, while at long times there exists an optimal value of x that allows to obtain the maximal density in the fraction of the system considered for the measurement. This phenomenology has been shown to be linked to the presence of heterogeneities in the system [4].

First case: $x_1 > x_2$. When, at t_w, x is suddenly lowered, the compaction rate of the perturbed system first increases. This behaviour, shown in figs. 2 and 3, is in agreement with the experimental results, and it is opposed to the results one would expect looking at the compaction data at constant forcing. This shows that the system has some memory of its history at t_w. After a transient, however, this memory is lost: compaction slows down, and the rate of compaction crosses over to the one observed at constant forcing: the curves of the reference and perturbed systems therefore cross.

While the transient, corresponding to the short-term memory, is observed for all values of the parameters investigated, an important difference appears for different t_w at long times: if t_w is small, the system is not very compact, and is able to evolve a lot during Δt. The system does not display memory for times larger than $t_w + \Delta t$. On the other hand, if the system is already quite compact at t_w (*i.e.* if it is *aged* enough), the system evolves very slowly during Δt, and for $t \geq t_w + \Delta t$ the compaction curves can be translated and superimposed to the reference one, as shown in the inset of fig. 3. During Δt the system is able to keep memory of its state at t_w. This shows that a sufficiently aged system displays a long-term memory, in addition to the short-term memory existing at all times.
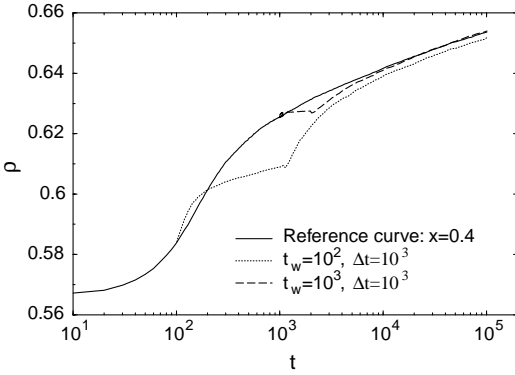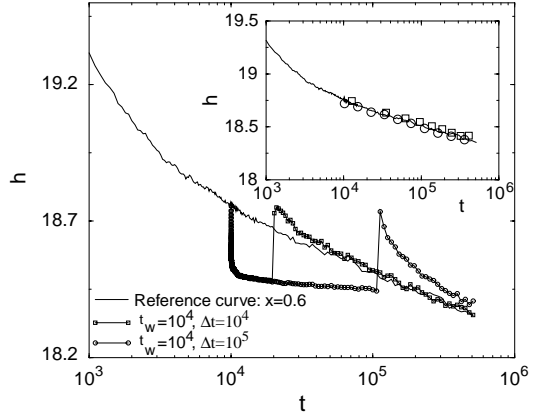
Fig. 2

Fig. 3

Fig. 2 – Density *vs.* time, for a reference case with constant $x = 0.4$, and with $x_1 = 0.4$, $x_2 = 0.1$, for various values of t_{w} and Δt. The first effect at t_{w} is an acceleration of the compaction, but at larger times the curves cross; after $t_{\mathrm{w}} + \Delta t$ the compaction is again fast. Size of the system: 60×60.

Fig. 3 – Effect of a decrease of the forcing with $x_1 = 0.6$, $x_2 = 0.3$, $t_{\mathrm{w}} = 10^4$, and $\Delta t = 10^4$ and 10^5: at short times after t_{w} the mean height h decreases strongly (corresponding to an increase in the compaction, as in fig. 1), but then remains almost constant. As x is again increased, h goes back to its value at t_{w}. The long-time memory effect is illustrated in the inset: the symbols, corresponding to the data for $\Delta t = 10^4$ and $\Delta t = 10^5$, are shifted by Δt and they coincide with the reference curve. Size of the system: 60×60.

The interpretation of these results is quite straightforward using the results of [4] and looking at the density profiles along the vertical direction: when x is abruptly lowered, the first effect is that the particles tend to go down, and the interface becomes steeper and more compact. Therefore the density first increases with respect to the unperturbed case. At larger times, however, the evolution is slowed down by the creation of a dense layer at the interface, which blocks the bulk rearrangements needed for the compaction [4]. After $t_{\mathrm{w}} + \Delta t$, the increase in the forcing allows to suppress the dense layer, and the compaction can become again fast. Moreover, if t_{w} is large enough (typically 10^3 or 10^4 MC steps), the bulk of the system is already quite compact, and therefore the smaller value of the forcing during Δt leads to a compaction of the interface but the bulk almost does not evolve. At $t_{\mathrm{w}} + \Delta t$, the forcing is again increased: since the relaxation of the interface is fast, this leads the system back to its state at t_{w}. The inset of fig. 3 illustrates this behaviour: the curves of compaction after Δt, shifted backwards by Δt, superimpose with the reference curves. This is not the case for lower values of t_{w}, for which the bulk is not yet highly compact at t_{w}, and evolves a lot even with a small driving x_2.

This memory effect, similar to what can be obtained in spin-glasses when the temperature is first lowered, then increased back to its previous value, was unfortunately not investigated in [6]: it can indeed occur only at long times. It would certainly be very interesting to have experimental checks on this aspect.

Second case: $x_1 < x_2$. When $x_1 < x_2$ we observe a short-term memory effect, as in the previous case. First, as the forcing is increased, one observes a decompaction; later on, the fact that x is larger prevails and the compaction proceeds faster, at the normal rate for constant $x = x_2$. The memory of the history up to t_{w} is lost after a transient (see fig. 4). These features
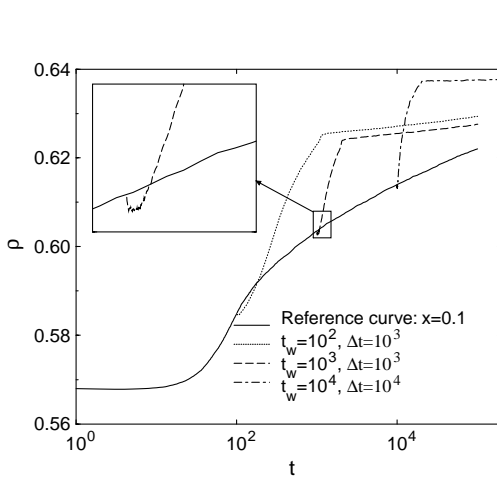
Fig. 4

Fig. 5

Fig. 4 – Effect of an increase in x (*i.e.* $x_1 < x_2$), for $x_1 = 0.1$, $x_2 = 0.4$, and various values of t_w and Δt; the inset shows a zoom of the decompaction effect at short times after the change in x. The compaction rate is then increased during Δt, and decreases again after $t_w + \Delta t$. Size of the system: 60×60.
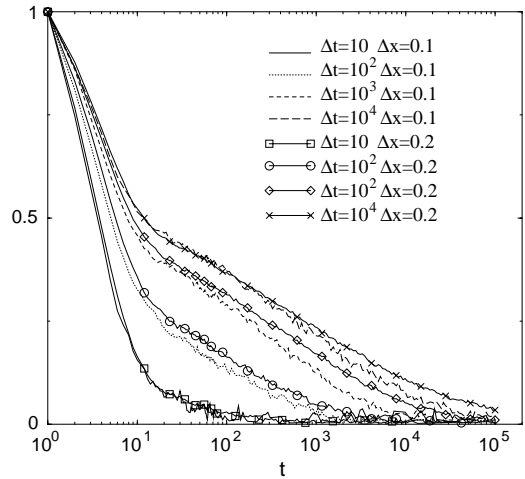
Fig. 5 – Relaxation of the potential energy of the system after a perturbation $\Delta x = 0.1$, 0.2 during Δt of a stationary state at $x = 0.1$ obtained after a slow decrease of the vibration intensity, starting from large forcing. Size of the system: 60×60.

are again similar to the experimental results. Moreover, at long times no memory is kept, since the whole system evolves a lot during Δt.

Once again, the study of the density profile allows for an interpretation of these results. As the tapping intensity is increased, the first effect is a decompaction, especially at the interface. The fact that the interface is less compact then allows for a much better compaction of the bulk. Note that this behaviour is closely related to the existence of a response function that is positive at short times and negative at longer times [4]. At $t = t_w + \Delta t$, the bulk has been deeply modified, so the system cannot have any memory of its configuration at t_w.

As a first partial conclusion we see that the interplay between the heterogeneities of the system and the influence of the forcing on different zones of the sample [4] leads to two quite different memory effects. On the one hand, the short-time memory is always present and it occurs in a way which is counterintuitive with respect to the effect expected on the basis of the behaviour at constant x. On the other hand, the long-time memory only exists for a cycle when $x_2 < x_1$ and for long times, *i.e.* for aged systems.

Perturbation of a time-translational invariant state. The numerical experiments mentioned above have been performed during the compaction process, *i.e.* while the system ages. Let us now focus instead on the behaviour obtained after a slow decrease of the vibration intensity, starting from large forcing: this is the usual way to obtain a very compact system [10] which, under weak driving x_1, is now in a state where time-translation invariance holds [4]. This state can now be perturbed by applying a stronger forcing x_2 for a time interval Δt, and the relaxation can be studied when the forcing is set back to the original value x_1. In [6], such experiments were performed for small Δt, showing non-exponential relaxations, the slower the larger is Δt. We are able here to explore different regimes, varying Δt over four orders of magnitude.

One observes a relaxation occurring in two steps (see fig. 5). First, one has a rapid decrease (during typically 10 time steps) corresponding to the relaxation of the topmost layers of the interface; this can be checked by looking carefully at the density profiles during the relaxation. Later on a much slower, almost logarithmic, decay occurs corresponding then to the re-compaction of layers under the interface. As Δt grows, deeper and deeper layers get involved during the decompaction process; as x is lowered again, the first relaxation, which compactifies the interface, makes it more difficult for the lower layers to get as compact as before: the relaxation therefore slows down.

Although the very dense medium, if shaken at constant x, is in a stationary state, the perturbation at larger x leads to the same mechanisms that are also at work during ageing. Therefore, it is not so surprising to find a slow re-compaction process after the perturbation.

The precise form of the relaxation can be expected to be model dependent, at least at short times: for example, in the case of "tapping", the first, rapid part of the relaxation does not exist. Therefore, we do not investigate it in detail, and underline only the most relevant features: even though the system seems in "equilibrium" (as shown in [4], the dense system is stationary and does not display ageing), the relaxation is non-exponential and very slow.

In summary, we have shown that the Random Tetris Model reproduces the complex phenomenology appearing in real experiments for large and abrupt changes of the driving: at small times after the change, the reaction of the system is opposite to what could be expected looking at the constant x compaction curves: if x is decreased, the system compactifies better, while it gets decompactified for an increase in x. At larger times, the compaction rate crosses over to the one at constant x: this effect was therefore called "short-term memory" [6].

Moreover, we have investigated the return to the previous value of the driving, in various time regimes, and shown that an increase in x erases quickly the memory of the previous configurations while, at long times, a decrease in x can "quench" the system until the previous value of x is restored and the compaction starts again from a configuration statistically equivalent to the one perturbed at $t = t_{\mathrm{w}}$.

It is noticeable that such a behaviour shares some similarities with the phenomena of rejuvenation and memory observed in the context of spin-glasses [5]. In temperature-cycling experiments on the ageing of spin-glasses indeed, the temperature is changed from T to $T + \Delta T$ at t_{w}, and back to T at $t_{\mathrm{w}} + \Delta t$; at t_{w} a restart of the relaxation is observed, and, at $t_{\mathrm{w}} + \Delta t$ and if ΔT is negative, a memory of the previous ageing at T is recovered. These effects have been and are still widely discussed and studied; they are certainly due to very subtle and complicated phenomena, and can be reproduced, *e.g.*, by hierarchical models of phase space [11], or by droplet-like models [12]. In the case of the RTM, we have seen that the explanation is much simpler, so that the analogy with spin-glasses seems only phenomenological and should certainly not be taken too far; the similarity of some results between such different physical systems (non-thermal granular media and thermal spin-glasses) is however interesting, in so far as it explains why models that have been used to describe the evolution of spin glasses in phase space (*e.g.*, trap model [13, 14] and random-walk model [7]) are also able to account for the phenomenology of granular compaction.

It seems to us, however, that the study of the heterogeneities (and therefore of real space models) could be crucial for the understanding of the basic mechanisms underlying the dynamics of these systems. In this respect, detailed experiments about the local rearrangements during compaction could provide valuable information for the understanding of the basic mechanisms at work in dense granular media.

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